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The Efficient Disequilibrium: A New Theory of the Business Cycle

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ABSTRACT: This study proposes a revolutionary theory of business cycles, arguing that they are not market failures but rather endogenous and essential for maximizing economic efficiency. To demonstrate our theory, we introduce the Augmented National Income Equilibrium Model Business Cycle, which comprises income, consumption, investment, the pure interest rate (R), and the potential return on investment (PRI). Through heuristic application of the model, we demonstrate how the dynamic interplay of these components generates business cycles that continuously clear without achieving traditional equilibrium. Our model shows that (i) the Central Growth Rate (CGR), which represents the long-term income growth trend, establishes the upper and lower bounds for the oscillatory relationship between changes consumption and investment when considered independently of the absolute income level, ensuring long-term stability within cyclical dynamics; (ii) R, which represents the time preference for consumption, closely tracks the CGR, mediating income allocation between consumption and investment; and (iii) continuous oscillation of the PRI-R Differential, which we define as $(PRI - R)$, is the primary driver of the business cycle, dynamically guiding capital allocation decisions.

Our study empirically confirms that oscillations in consumption and investment fully account for GDP cyclicality. While shocks and other destabilizing influences undoubtedly play a role when they occur, the Augmented NIE Model Business Cycle demonstrates that cyclical behavior emerges even in their absence. When we remove the influence of the absolute level of income on consumption

This working paper represents the formal development of a theory that has informed my market analysis for nearly 40 years. The initial ideas were conceived around the time of my book, *The Speculator's Edge*, in 1989, during my career as a professional speculator. Comments and feedback are welcome at a.p.pacelli@gmail.com.

and investment by normalizing them, the model reveals an inverse relationship between the two, making it impossible for their relative shares of income to grow simultaneously at the same rate, a condition necessary for steady-state income growth. Similarly, removing the influence of the absolute level of consumption on income and investment reveals their equality, demonstrating that the economy efficiently allocates income growth to balance the immediate demand for consumption with investment for greater future growth. The Augmented NIE Model Business Cycle also shows that in acting as a boundary governing the peaks and troughs of income-normalized consumption (dC^*/dt) and income-normalized investment (dI^*/dt), the Central Growth Rate (CGR) imposes cyclical regularity on the oscillations in the relative income shares of the two inversely related income components, resulting in the persistent cyclical activity that characterizes business cycles.

The insights offered by the Augmented NIE Model Business Cycle carry profound implications for economic policy, suggesting that interventions aimed at dampening or accelerating natural business cycle oscillations may inadvertently disrupt the economy's self-regulating efficiency.

1 Introduction

Few modern economic phenomena receive more attention than business cycle theory. Business cycles stick out like sore thumbs in successful capitalist economies such as ours, which has produced almost unimaginable wealth. At their mildest, business slowdowns cause increased unemployment and anxiety. When severe, they may yield poverty, starvation, and political upheaval.

In economic theory, the absence of a satisfactory explanation for persistent, cyclical business activity also sticks out like a sore thumb. Despite significant attention, only a few business cycle theories have achieved wide acceptance, and none holds preeminent status.[1]

The term "business cycle" typically refers to the fluctuation of aggregate economic activity from expansion (GDP growth greater than or equal to zero) to contraction (GDP decline) and back again. During the expansion phase, economic activity, generally measured by the dollar value of transactions, increases and remains positive. The expansion phase may also include rising

[1] Indeed, the debate as to whether business cycles must occur (i.e., are endogenous) continues. We will discuss the most widely acknowledged business cycle theories in Section 2.

employment, wages, interest rates, and prices. Conversely, the contraction phase may witness declines in these categories.

Although real income gains and full employment are crucial to individuals, the business cycle is a phenomenon of aggregate economic activity and must be measured and understood at that level. This paper seeks to demonstrate that the business cycle is a disequilibrium phenomenon resulting from changing valuations of income saving and investment[2] versus consumption.[3] According to our theory, an economy characterized by secular growth will always have either excessive investment and insufficient consumption or excessive consumption and insufficient investment for output stability. Therefore, although we accept the traditional definition of "business cycle," our focus will be on the continuous and alternating adjustment process of income investment and consumption as its defining structural characteristic.

Under our theory, the rate of change of economic output will oscillate around the rate of secular output growth, which we will call the Central Growth Rate (CGR). Our presentation will sometimes consider a model economy in which the CGR is persistent and steady within and across business cycles, which we will call a continually progressing economy (CPE). However, in real-world economies, there is little evidence for such to be the case for any extended period, and, as will be readily apparent from our discussion, changes in the CGR will necessarily alter the course of economic activity via their impact on the potential return on investment (PRI) and the market price for loanable funds, as a separate but non-essential causal phenomenon. Nevertheless, we will show that endogenous changes in the valuation of income investment and consumption must result in cyclical economic activity even in our CPE model, which would offer the best prospect for non-cyclical steady growth, and it follows that the dynamic valuation process will produce cyclical economic activity in all CGR conditions.

Although equilibrium growth at the CGR or cyclical "soft landings," in which the business cycle troughs at or near zero growth, are often the goal of policymakers and participants alike, the former is impossible, and the latter is unlikely without disruption of the CGR or "hands-

[2]For the remainder of this paper, the term "investment" will be used instead of "savings and investment" unless the context otherwise requires.

[3]For foundational discussion on consumption and investment dynamics, see [30], Chapters 7–8.

off" economic policy. Even when output oscillates around a steadily positive *CGR*, its troughs will most often fall below zero growth. Fluctuating wages, prices, interest rates, and some level of unemployment, which mark all business cycles, may represent frictions (barriers or delays in the efficient reallocation of resources) that inhibit rapid adjustments of income, causing negative output. In this regard, consider that while prices of many commodities adjust to changes in supply and demand instantaneously, wages and employment do not. In the case of labor, the natural resistance due to labor market norms and contractual expectations of employees to accept reduced wages may necessitate layoffs, which occur with a lag, take time to work through the economy, and may result in a more significant reduction of purchasing power.[4]

A *CPE* would require investment and consumption to grow alongside income at the *CGR* at a constant return on investment. However, as will be explained in detail in Section 5 (*The Impossibility of a Steady-State Equilibrium*), natural economic dynamics render simultaneously equal growth rates of income, consumption, and investment impossible. Within each business cycle, there is a phase in which the availability of profitable projects and loanable funds for investment in them will enable the "pulling forward" of future growth at an above-*CGR* rate. After this phase plays out with the maximization of the growth rate, an opposing phase emerges in which consumption's share of income increases, and investment and income decline below trend. The two phases result in mean growth at the *CGR*. The alternating dominance of investment and consumption during these two phases progressively repeats from one business cycle to the next. Unless one assumes that central planners are better than the free market at making investment decisions, any intervention will likely result in an inefficient reallocation of income and wealth. Because business cycles assist in the most efficient allocation of the free market's resources, even if it is possible to avoid them, it is not economically desirable to attempt to do so,[5] except perhaps in extreme cases in which the

[4] Numerous studies have demonstrated the persistence of wage rigidity and its adverse macroeconomic consequences. See [26] for a comparative study across 53 countries. For country-specific examples, see: [1] (nominal wage rigidity sustained unemployment in Sweden's 1990s economic crisis); [16] (research comparing union and non-union wages in the U.S. found that the former are more rigid than the latter); and [10] (comparative research in Germany and Italy highlighted prolonged employment challenges caused by real wage rigidity).

[5] Our intention is not to deny that there may not be political or humanitarian policy reasons for government interference in the free markets; however, we assert that any such activity inevitably entails a redistribution of income and wealth at an overall cost to the economy's participants, which should be openly acknowledged.

stability of the system itself is in doubt.[6]

Because the business cycle is not a monetary phenomenon but a consumption-investment phenomenon at its core, it does not even require the existence and widespread use of money for transactions, which is a fact that pure monetarists cannot overcome. Although monetary-policy-driven interventions can significantly impact growth and prices by directly reallocating resources, such interventions suffer from the same infirmities as other forms of intervention.

The Augmented National Income Equilibrium Model Business Cycle presented in this paper encapsulates a novel theory of business cycles. The model builds upon the National Income Equilibrium (NIE) model which arose out of Keynesian theory and comprises income (Y), consumption (C), investment (I). We augment the NIE Model to include the pure interest rate (R) and the potential return on investment (PRI) and refer to the augmented model as the "Augmented NIE Model" and the five variables included in it as the "Augmented NIE Augmented NIE Model Variables". The Augmented NIE Model presents a static structure to show the relationships among the Augmented NIE Model Variables. The Augmented NIE Model Business Cycle shows how these relationships operate dynamically to produce business cycles that continuously clear without ever achieving equilibrium.

The Augmented NIE Model Business Cycle posits that business cycles are endogenous and efficient and that although a "natural rate" exists, it is dynamic, emerges naturally from endogenous elements, and is not part of a stable income equilibrium. In this paper, we will first introduce the model and its key concepts, then contrast it with other traditional business cycle theories, including equilibrium-based approaches and those focused on monetary or innovation-based drivers of the business cycle. Next, we will lay out the formal mathematical framework of the Augmented NIE Model, and discuss the model's central growth proposition, the definition of a recession, and the role of time preference as an income allocation mediator. We will then demonstrate the impossibility of a steady state continually progressing economy and describe the cyclical phases of investment and

[6]One needs to look no further than the banking bailouts that accompanied the Great Recession for an example in which intervention appears to have been necessary to save the system; however, one should also consider whether the Great Recession itself was an unnecessary consequence of poor fiscal and monetary policy. See, for example, [15] (arguing that government intervention and expansionary monetary policy caused the boom and bust) and [4] (critiquing mainstream views on the financial crisis and arguing that mismanagement of the money supply caused the recession).

consumption inherent within each business cycle. After that, We will present empirical evidence for the validity of the underlying components of the Augmented NIE Model Business Cycle, synthesize those components into the complete model, and demonstrate how the model integrates the income, investment and consumption dynamics of a theoretical economy, appying it to real-world U.S. data from 1962 to 2024. Finally, we will draw upon the Augmented NIE Model Business Cycle present its major policy implications.

2 Our Disequilibrium Model in Context

Traditional business cycle theories generally operate under the assumption that economies naturally gravitate towards a state of stable equilibrium. These theories pinpoint disruptive factors—such as monetary shocks, technological innovations, or shifts in aggregate demand—to account for observed economic cycles. Prominent examples of these equilibrium theories include: Pure Monetary Theory, which focuses on money supply; the Monetary Over-Investment Theory, which explains cycles through credit expansion; Schumpeter's Theory of Innovation, which emphasizes technological disruptions; Keynes's Theory, which introduced the concept of aggregate demand; Samuelson's Model of Multiplier Accelerator Interaction, which marries multiplier effects with investment cycles; and Hicks's IS-LM model, which provides a framework for analyzing interest rates and economic output. These theories collectively suggest that without exogenous shocks or policy errors, economies would maintain a steady state, in stark contrast to our model's premise that cycles are an inherent result of income allocation dynamics between consumption and investment.

One particularly influential concept in modern equilibrium theory is the natural interest rate, often denoted as *r-star*, which represents the real (inflation-adjusted) interest rate theoretically consistent with an economy operating at full employment and stable inflation.[7] Proponents of *r-star* consider it a neutral rate because it aligns the economy's aggregate demand with its potential output, avoiding any unnecessary stimulation or hindrance of economic growth. Economists argue that when real rates are below *r-star*, they are expansionary, encouraging borrowing and investment, and that real rates above *r-star* are contractionary, discouraging these activities. Advocates of *r-*

[7] See [19] for a discussion of the challenges of measuring the natural rate of interest.

star further argue that stable economic growth can be supported by maintaining the natural rate of interest close to *r-star*.[8] While we are critical of models that seek to impose a *static* natural rate of interest in pursuit of a steady-state economy, we acknowledge the logic of the concept of a natural interest rate (R) that aligns with the Central Growth Rate (CGR). Indeed, we propose such an interest rate as a mediator in the dynamic allocation of income between consumption and investment.[9]

In contrast to the equilibrium-based approaches, a growing body of literature posits inherent disequilibrium within free-market economies. Prominent disequilibrium theorists include Ludwig Lachmann[10], Hyman Minsky[11], and Axel Leijonhufvud[12], who have explored the role of uncertainty, financial instability, and coordination failures as endemic economic factors generating cyclical business activity.[13] More recent contributions have sought to refine these ideas. Vianna (2023) discusses how business cycle theories following Keynes have evolved, particularly in their treatment of equilibrium and instability, offering an updated critique of traditional models.[14] Similarly, Gross (2022) provides an alternative perspective on endogenous business cycles, emphasizing interest rate perturbations as a key mechanism of disequilibrium.[15] Finally, Anundsen et al. (2014) examine endogenous cycles within disequilibrium models, highlighting the role of financial constraints and overdeterminacy in shaping economic fluctuations. While their work acknowledges endogenously driven cycles, it differs from our approach in viewing these cycles as deviations from equilibrium rather than an efficient feature of economic dynamics.[16]

Our theory adopts a disequilibrium perspective but diverges from existing disequilibrium theories in several significant ways. Unlike many theorists who focus on factors like uncertainty or

[8] See [29] and [11].

[9] The heuristic Augmented NIE Model Business Cycle shown in Figure 19, which depicts a constantly progressing economy (CPE) with a steady CGR, shows R in perfect alignment with the CGR while the PRI and the rates of change in income (dY/dt) and investment ($dI^{\#}/dt$) oscillate around it; however, while R is the most stable of the variables shown in Figure 19, Figure 24, which depicts $dC^{\#}/dt$ (our proxy for R), shows that in the real-world R is never static.

[10] [18], pp. 17-23

[11] [27], pp. 20-27

[12] See [21], particularly Part II, for an exploration of effective demand failures and the conditions under which self-regulating markets may fail to establish coordination.

[13] For a broader overview of disequilibrium theory, see [22].

[14] [33]

[15] [13]

[16] [2]

financial instability, we argue that the core driver of the business cycle is the ongoing tug-of-war between investment and consumption of income, mediated by the pure interest rate (R), which reflects the time preference for consumption.[17] By establishing the minimum return threshold that potential investments must achieve, R governs the allocation of income between consumption and investment.[18] Although we acknowledge that elements highlighted by equilibrium and other disequilibrium theorists can impact this interaction, we consider these elements modifiers rather than primary causes.

3 Preliminary Matters

Before commencing analysis, we should state our assumptions and qualifications and provide additional definitions. The NIE Model that lies at the heart of our analysis states:

$$Y = C + I, \tag{1}$$

where Y represents aggregate national income, C represents aggregate national consumption, and I represents aggregate national investment.

In constructing our Augmented NIE Model, we add R and PRI to the basic NIE model as crucial determinants of investment (I). The interaction between the supply of loanable funds, made available by agents willing to defer consumption, and the demand for such funds for investment purposes, driven by PRI, determines R, the theoretical interest rate that arises in a loanable funds market. For the loanable funds market to be in equilibrium at any moment, current income or additional borrowing in anticipation of positive investment returns must be adequate to meet the cost of borrowing. R and PRI interact dynamically, continuously adjusting to balance the allocation of income between investment and consumption within and across business cycles. The

[17] See [30], Chapter 7, for a discussion of consumption theories and their implications for macroeconomic dynamics.

[18] In the context of our Augmented NIE Model Business Cycle of a continually progressing economy (CPE), we use the term "mediate" to describe R's role in determining the allocation of income between consumption (C) and investment (I). Although R itself remains stable in our model, it acts as a "gatekeeper", ensuring that consumption remains optimized both during and across business cycles. Specifically, R represents the threshold return that investment opportunities must exceed to be considered viable alternatives to immediate consumption. Thus, R serves as a stable "pivot point" around which the dynamic allocation of income (Y) between C and I occurs, effectively mediating the ongoing balance between these two fundamental economic activities.

inclusion of R and I in the Augmented NIE Model is reflected in Equation (2) ($PRI = R + dI^{\#}/dt$), which ties them to I.

Economists have proposed various interest rates as proxies for R, but none have proven entirely satisfactory. Based on our analysis of the Augmented NIE Model Business Cycle, we propose (in Section 7.7) a new theoretical framework for R, incorporating the growth rate of income-normalized consumption as a credible candidate for income-normalized R (R^*) and marginal income-share adjusted consumption, $dC^{\#}/dt$ for R.[19] We recommend this approach as a promising avenue for further study of the dynamic interplay between consumption, investment, and income.

PRI represents the potential return on investment after deducting all production costs without considering R (the cost of borrowing). The importance of PRI in the analysis of the Augmented NIE Model lies in its relationship with R in determining the level of investment (I) at any given time. We can express this relationship as:

$$PRI = R + \frac{dI^{\#}}{dt}, \tag{2}$$

where $dI^{\#}/dt$ is the marginal, income-share adjusted investment borrowed and applied to income production, expressed as a rate of change over time. The dynamic interaction between R and PRI reflected in $dI^{\#}/dt$ is central to understanding the cyclical nature of investment and consumption in our disequilibrium model.

To focus on core cyclical dynamics, we abstract from the full National Income and Product Accounts (NIPA) model ($GDP = C + I + G + NX$), simplifying our analysis by excluding government spending and net exports, except to the extent that they are indirectly reflected in consumption and investment.[20] While no real-world economy operates in complete isolation, and even the U.S. is deeply integrated into global trade and financial markets, we contend that for economies where private consumption and investment constitute a substantial portion of overall economic

[19]We will employ the asterisk ("*") to designate variables we have normalized for income and the hashmark ("#") to designate unnormalized variables that we have adjusted for their share of income, as explained in detail below.

[20]To be clear, we are not asserting that U.S. GDP (or the GDP of any other nation) is solely a function of consumption (C) and investment (I), rather that our model's focus on C and I is sufficient to illustrate the outcomes predicted by our theory.

activity, the Augmented NIE Model Business Cycle provides a robust framework for analyzing business cycles. By focusing on private economic activity in the U.S., our object of study, the model sufficiently captures the primary economic dynamics necessary to analyze business cycles as a disequilibrium phenomenon, with a particular focus on the role of R in optimizing consumption through its mediation of income allocation. This assertion is strongly supported by the exceptionally high correlation observed in our time series data, as illustrated in Figure 1 (*Correlation of GDP Proxy to GDP*) ($R^2 = 0.9995$) and Figure 4 (*Normalized Consumption and Investment Derivatives*) ($R = -1$).

In considering investment (I), we are not interested in reborrowing funds already invested or borrowing funds for consumption because such transactions merely transfer consumption without changing its amount. Neither do we need to inquire into the conditions for secular growth, which, of course, may include increases in population, innovation, discovery of new resources, and others because the subject of our inquiry is secularly growing economies regardless of their underlying growth engines. Instead, our focus will be exclusively on what economic agents do with the marginal income produced by the economy and the choices of such agents between consuming or investing it during and across business cycles.

Finally, in characterizing a model economy as a *CPE* under the Augmented NIE Model Business Cycle, our expectation, supported by our analysis, is that as the *CPE* grows across business cycles, all three variables will tend to grow at or near the *CGR*.

4 Foundations of the Augmented NIE Model

4.1 Marginal Returns on the Augmented NIE Model Variables

In the context of our model, we can develop formal statements about the relationship between investment, consumption, and income, as follows:

1. MY or dY/dt: In this work, we will sometimes use MY (marginal income) as equivalent to dY/dt. dY/dt represents the marginal income produced by investment (and land and labor) and is understood as the marginal product of capital. MY represents the *potential* for either

consumption or investment.[21]

2. MC or dC/dt: represents the marginal utility derived from consuming an additional unit of a good. In traditional economic theory, the marginal utility of consumption decreases as the quantity consumed increases, a principle known as diminishing marginal utility, which reflects that each successive unit consumed provides less additional satisfaction than the previous one. At an aggregate level, as overall income declines, the marginal utility of consumption relative to income to tends to increase because, with lower income levels, each additional unit of income spent becomes more crucial for satisfying basic needs and maintaining well-being, thereby becoming more valuable.[22]

3. MI or dI/dt: represents the marginal utility derived from investing an additional income unit. In economic theory, the marginal utility of investment often exhibits diminishing returns as the amount invested increases. At an aggregate level, as overall savings rise, the marginal utility of further investment tends to decrease because the potential return on marginal investment (PRI) declines, reflecting diminishing opportunities for high-yield projects, which become scarcer with incremental capital allocation.[23]

4. R: R represents the pure interest rate, functioning as the cost of borrowing and the reward for deferring immediate consumption. R reflects the price economic agents pay to access loanable funds or the benefit received for choosing to save rather than consume. In a stable, functioning economy, R is consistently positive, as some degree of compensation is required to incentivize saving and enable lending for investment purposes.[24]

5. PRI: PRI denotes the potential return on investment after covering all production costs but excluding R, the cost of borrowing. PRI reflects the maximum possible gain from investment opportunities within the economy and represents the demand for loanable funds based on

[21] See [25], Chapter 10, "Economic Growth," for a discussion of the marginal product of capital as a determinant of potential output.
[22] See [25].
[23] See [5].
[24] See [8], Chapters 1, 4, and 5.

expected investment profitability. PRI enables comparison with R to assess whether investment opportunities justify the deferral of consumption, determining the allocation of resources toward either consumption or investment.[25]

Each of MY, MC, MI, R, and PRI has the dimension of the reciprocal of time, effectively allowing each variable to be interpreted as an interest rate, reflecting how they quantify the pace of allocation, transformation, or valuation of economic resources.

4.2 Formal Relationships Among the NIE Model Variables

Before laying out the symbolic representation of the Augmented NIE Variables and some of their important relations, it will be helpful to clarify denotation. When we refer to absolute changes in raw variables without relation to time, we will use the Δ symbol (e.g., ΔY). If we refer to changes in raw variables temporally, we will designate the change as a derivative. Since we are dealing primarily with annual analysis of quarterly data, we treat percent change with respect to the applicable period as equivalent to a derivative (e.g., dC/dt means $100 \cdot \Delta Y/Y$). Because business cycles are equivalent to income cycles, most equations in this article refer to changes in the income-share adjusted Augmented NIE Variables, denoted with the hashmark "#" (such as $dC^{\#}/dt$, which means $(dC/dt) \cdot (C/Y)$). Finally, when we analyze changes in the income-share adjusted Augmented NIE Variables normalized for income, we denote those changes with the asterisk "$*$" (such as dC^*/dt, which equals $d(C/Y)/dt \cdot (C/Y)$).

It makes the most sense to begin our analysis with the NIE Model.[26] Under Equation (1) ($Y = C + I$), income equals consumption plus investment at any time. We can consider the changes over discrete periods rather than take a continuous derivative to identify what is occurring at the margin. This yields:

$$\Delta Y = \Delta C + \Delta I, \tag{3}$$

where ΔY, ΔC, and ΔI represent the raw changes in income, consumption, and investment, re-

[25] See [8], Chapters 6 and 7.

[26] Appendix C (*Summary of Augmented NIE Model Equations)* sets forth a comprehensive list of the major equations derived and used in this article.

spectively, over one period (e.g., one year).

Since our goal is to understand the oscillations in income across business cycles, we will primarily consider the rates of change of the Augmented NIE Augmented NIE Model Variables in terms of percent returns or interest rates. To express these changes in terms of annual percent growth similar to interest rates, which under our notation rules requires no special characters, we define:

$$\frac{dY}{dt} = 100 \cdot \frac{\Delta Y}{Y_{T0}}, \tag{4}$$

$$\frac{dC}{dt} = 100 \cdot \frac{\Delta C}{C_{T0}}, \tag{5}$$

$$\frac{dI}{dt} = 100 \cdot \frac{\Delta I}{I_{T0}}. \tag{6}$$

Here, Y_{T0}, C_{T0}, and I_{T0} are the initial values of income, consumption, and investment, respectively.

To determine the rates of change of the NIE Model Variables using the straightforward derivative methodology from traditional calculus, where income (Y) is the sum of consumption (C) and investment (I), the analysis would be:

$$\frac{dY}{dt} = \frac{dC}{dt} + \frac{dI}{dt} \tag{7}$$

However, because our methodology considers percent changes over measurable periods rather than absolute changes over infinitesimally small periods, this equation does not hold. Percent changes do not sum in the same way as traditional derivatives because they are relative to different bases (the initial values of Y, C, and I), and, in fact, the sum of dC/dt and dI/dt is equal to $2 \cdot dY/dt$, which is consistent with one of our fundamental premises that for sustained economic growth all three variables must increase by corresponding percentages over time. To correctly analyze percentage changes within the Augmented NIE Model , we adjust Equation (7) to account for the relative sizes of C and I within Y:

$$\frac{dY/dt}{Y} = \left(\frac{dC/dt}{C} \cdot \frac{C}{Y}\right) + \left(\frac{dI/dt}{I} \cdot \frac{I}{Y}\right) \tag{8}$$

This approach allows us to analyze the shares of consumption and investment in driving overall income growth. With this formulation, we can rigorously determine the portions of income growth that are accounted for by consumption growth and investment growth, ensuring that the analysis properly reflects the contributions of these components to overall economic dynamics. Using our notation, we can rewrite Equation (8) as:

$$\frac{dY}{dt} = \frac{dC^{\#}}{dt} + \frac{dI^{\#}}{dt} \tag{9}$$

The reader should note that even though we are dividing the change in income by the level of income in Equation (9), our notation convention does not require the "$*$" for income (Y) because it cannot be normalized for itself.

To best analyze the relationship between consumption and investment without regard to the level of income (i.e., without including the direct effect of changes in the level of income on the share of consumption or investment in income), we can normalize the two variables for income, effectively removing the influence of income from the NIE Model to reveal changes in their relative *share of income*. Although we will see that changes in income-normalized consumption and investment are closely and consistently associated with dY/dt and tend to cross above and below zero at the beginning and end of recessionary periods, for reasons addressed in the text below and in Appendix A (*Normalization and Unnormalization Methodology*) they do not provide sufficient information to determine the level of dY/dt at any time.

We can normalize consumption and investment in either of two ways. We can divide each variable by income before operating on it (e.g., dividing GPCE (our proxy for consumption) by the GDP Proxy (our proxy for income) and then taking its derivative and multiplying it by its income share to obtain income-normalized dC/dt (i.e., dC^{*}/dt)). Alternatively, when working with derivatives, we can subtract from the integral of the derivative the integral of the GDP Proxy for the corresponding period and then take the derivative of the difference and multiplying it by its income share before performing any analysis or other operations on it, ensuring that we account for income fluctuations consistently. This process yields outcomes equivalent to directly dividing each variable by income before taking its derivative. In either case, as noted, we will designate the income-normalized variable by appending the "$*$" superscript.

As it turns out, we will rarely need to refer to dC^*/dt or dI^*/dt on an unadjusted basis (relative to income). Solely in the case of those two variables and their corresponding terms (MC^*) and (MI^*), we will use the asterisk ("$*$") to mean "normalized for income and income-share adjusted" to account for both the rate of change of its income share and its proportional relationship to income.

When we normalize other variables, for example, by subtracting from the three-month U.S. Treasury bill rate (3MTB) the year-over-year change in income, we will employ the asterisk ("$*$") to reflect only normalization (e.g., $3MTB^*$).

To determine the income-normalized relationship between the rates of change of consumption and investment we can divide both sides of Equation (1) ($Y = C + I$) by Y to get:

$$1 = \frac{C}{Y} + \frac{I}{Y}$$

If we take the derivative of the result, we obtain:

$$0 = \frac{dC^*}{dt} + \frac{dI^*}{dt}$$

We can rearrange this result to arrive at the following:

$$\frac{dC^*}{dt} = -\frac{dI^*}{dt} \tag{10}$$

We readily see from Equation (10) that changes in consumption are inversely related to changes in investment.

At the individual level, due to personal preferences, the actual proportion of income available for lending is unlikely to coincide with the proportion for the aggregate economy. If an individual's time preference for consumption is greater than R, they will not be willing to defer consumption to lend any portion of their income at R. Conversely, if their time preference is lower than R, they will be more inclined to save and lend. However, we can provide a significantly more robust model for the aggregate economy because, unlike individual preferences for consumption and investment, which are subjective to each economic agent, the aggregate preferences of an ideal economy are objective and quantifiable by reference to actual market data.

As noted, for the aggregate economy, R, a rate set by the market, represents both the time preference for consumption and the cost of borrowing funds for investment. As such, at any moment in the business cycle, R represents the rate borrowers must offer to ensure the availability of funds for their investment. Therefore, the profitability of investment depends upon the availability of returns that exceed R, and we see that, at the margin, the PRI-R Differential determines $dI^{\#}/dt$. We can represent the relationship among R, PRI, and $dI^{\#}/dt$ as follows:

$$R = PRI - \frac{dI^{\#}}{dt} \tag{11}$$

which is a rearrangement of Equation (2). From Equation (11), we can see that when $PRI = R$, $dI^{\#}/dt = 0$.

We can normalize Equation (11) for income by subtracting dY/dt from R and PRI and replacing $dI^{\#}/dt$ with dI^{*}/dt as follows:

$$R^{*} = PRI^{*} - dI^{*}/dt \tag{12}$$

Finally, we can rearrange Equation (11) and substitute the PRI-R Differential into Equation (9) to get:

$$\frac{dY}{dt} - \frac{dC^{\#}}{dt} = PRI - R,$$

and, rearranging again, get:

$$PRI - R = \frac{dY}{dt} - \frac{dC^{\#}}{dt} \tag{13}$$

Equations (9), (10), and (11) will be important when demonstrate the impossibility of a steady-state *CPE* in Section 5 (*The Impossibility of a Steady-State Equilibrium*) and we construct our

Augmented NIE Model Business Cycle in Section 7.7 (*Building the Augmented NIE Model Business Cycle*).

4.3 The Central Growth Proposition

4.3.1 Proof of the Central Growth Proposition.

The Central Growth Proposition (CGR Proposition) states:

$$CGR = \frac{dY}{dt} + \frac{dC^*}{dt}$$

We can use the relations among the NIE Model Variables to prove the CGR Proposition based on the assumption that $C/Y \approx 1$:[27]

Step 1: Start with the definition of income-normalized consumption ($dC^*/dt = d(C/Y)/dt \cdot C/Y$) and expand it using the quotient rule to get:

$$\frac{dC^*}{dt} = \left[\frac{\frac{dC}{dt} \cdot Y - C \cdot \frac{dY}{dt}}{Y^2}\right] \cdot \frac{C}{Y}$$

Step 2: Simplify the expression:

$$\frac{dC^*}{dt} = \frac{\left(\frac{dC}{dt} \cdot Y - C \cdot \frac{dY}{dt}\right) \cdot \frac{C}{Y}}{Y^2}$$

Step 3: Apply the approximation $C/Y \approx 1$:

$$\frac{dC^*}{dt} \approx \frac{\frac{dC}{dt} \cdot Y - C \cdot \frac{dY}{dt}}{Y^2}$$

[27] Our assumption that $C/Y \approx 1$ is supported by empirical analysis of the full study period (1962 Q1 - 2024 Q3). The actual mean dY/dt is 6.76% whereas the model-derived CGR ($dY/dt + dC^*/dt$) averages 6.83%. The variance between the series remains constrained to ±0.5% in 95% of observations. Standard deviation analysis confirms 68% of cases fall within ±2.6% of 6.76%, and 99.7% remain within ±7.8%. This tight correlation demonstrates that although C/Y averages 0.9246, the simplifying assumption that $C/Y = 1$ introduces less than 0.07% mean error in CGR estimates while preserving model integrity.

Step 4: Multiply both sides by Y to relate the expression to our core terms:

$$\frac{dC^*}{dt} \cdot Y \approx \frac{dC}{dt} - \frac{dY}{dt}$$

Step 5: Rearrange the terms:

$$\frac{dC^*}{dt} + \frac{dY}{dt} \approx \frac{dC}{dt}$$

Step 6: Apply the CGR Proposition ($CGR = dY/dt + dC^*/dt$):

$$CGR \approx \frac{dC}{dt}$$

Step 7: Given that $dC^\#/dt \approx dC/dt \cdot C/Y$, and our assumption that $C/Y \approx 1$, we can conclude:

$$CGR \approx \frac{dC^\#}{dt}$$

This result confirms that under our assumptions, we can indeed approximate the Central Growth Rate (CGR) by the sum of the rate of change of income (dY/dt) and the rate of change of income-normalized consumption (dC^*/dt), thus proving our proposition:[28]

$$CGR = \frac{dY}{dt} + \frac{dC^*}{dt} \approx \frac{dC^\#}{dt} \tag{14}$$

4.3.2 Demonstrating that CGR is the Mean of dY/dt as it Oscillates.

In this subsection, we will demonstrate that the Central Growth Rate (CGR) represents the mean of dY/dt as it oscillates within and across business cycles. This follows directly from the CGR Proposition ($CGR = dC^*/dt + dY/dt$).

Step 1: Rearrange the CGR Proposition to express dY/dt in terms of the CGR and dC^*/dt:

$$dY/dt = CGR - \frac{dC^*}{dt}$$

[28]The justification for equating $dC^\#/dt$ with dC/dt is identical to the justification of our simplifying assumption. See footnote 27.

Step 2: Integrate both sides over a complete cycle (T) to determine the mean behavior of dY/dt:

$$\int dY/dt\,dt = \int (CGR - \frac{dC^*}{dt})\,dt$$

Step 3: Split the integral:

$$\int dY/dt\,dt = \int CGR\,dt - \int \frac{dC^*}{dt}\,dt$$

Step 4: Because in the Augmented NIE Model Business Cycle, CGR is stable over the cycle[29] its integral simplifies to:

$$\int CGR\,dt = CGR \cdot T$$

By the Zero-Sum Dynamic of the Augmented NIE Model , the oscillations of dC^*/dt balance perfectly, meaning:

$$\int \frac{dC^*}{dt}\,dt = 0$$

Step 5: Substitute these values back to get:

$$\int dY/dt\,dt = CGR \cdot T - 0$$

or equivalently:

$$\int dY/dt\,dt = CGR \cdot T$$

Step 6: Divide through by the length of the period T, to obtain the mean of dY/dt:

$$\frac{1}{T}\int dY/dt\,dt = CGR$$

Thus, over a complete period, the mean of dY/dt equals the Central Growth Rate (CGR).

Step 7: This result implies that the oscillations of dY/dt above and below CGR balance perfectly,

[29] This proposition follows from the fact that the model depicts a CPE that grows at a constant rate.

as expressed by:

$$\int (dY/dt - CGR)\, dt = 0$$

or equivalently:

$$\int dY/dt > CGR = \int dY/dt < CGR$$

This demonstrates that CGR serves as the equilibrium line around which dY/dt oscillates, reinforcing its structurally vital role in the Augmented NIE Model Business Cycle.

4.4 The Recession Condition.

For modeling purposes, we can define a recession as any time in which $dY/dt < 0$. We can use Equation (14), the CGR equation, to restate the recession definition in terms of dC^*/dt and dI^*/dt.

Step 1: Rearrage the CGR equation as follows:

$$\frac{dY}{dt} = \text{CGR} - \frac{dC^*}{dt}$$

Step 2: Insert the recession condition ($dY/dt < 0$) into the CGR equation:

$$\frac{dY}{dt} = \text{CGR} - \frac{dC^*}{dt} < 0$$

Step 3: Restate the Identity: We can restate the identity in terms of $dC * /dt$:

$$\frac{dY}{dt} < 0 = \frac{dC^*}{dt} > \text{CGR}$$

Step 4: Apply the Zero-Sum Dynamic ($dC^*/dt = -dI^*/dt$) to restate the recession condition in terms of $dI^/dt$:

$$\frac{dY}{dt} < 0 = \frac{dI^*}{dt} < -\text{CGR}$$

This demonstration tells us that the income growth rate (dY/dt) will be negative when the income-normalized consumption growth rate (dC^*/dt) is greater than the CGR and the income-normalized investment growth rate (dI^*/dt) is less than $-CGR$.

4.5 Time Preference as the Income Allocation Mediator

In classical economic models, such as those developed by Irving Fisher, time preference is a significant but not sole determinant of interest rates. Time preference reflects individuals' trade-offs between present and future consumption based on their marginal rate of substitution. However, interest rates are also influenced by investment opportunities and the productivity of capital. Individual time preferences are not fixed but rather adjust dynamically to the market rate of interest through borrowing and lending. The equilibrium interest rate is thus the point at which subjective time preferences and objective investment opportunities are balanced.[30]

The Augmented NIE Model Business Cycle offers a significant departure from these classical theories by demonstrating that the time preference, R, is an endogenous outcome of business cycle dynamics rather than an exogenous determinant of interest rates. While Fisher's framework emphasizes individual time preferences as a primary influence on interest rates, with investment opportunities playing a role, the Augmented NIE Model Business Cycle presents R as the dynamic equilibrium price mediating the allocation of income between consumption and investment. Unlike Fisher's more static view, in which time preference serves as a driver of the interest rate, this framework reveals that R is dynamically determined by the economy's structural growth rate and actively regulates the allocation of resources over time.

We can establish the role of R in mediating income allocation through a series of interconnected proofs that demonstrate how R constrains the relative movements of income and investment and governs capital allocation decisions. These proofs validate the model's assertion that R is neither arbitrary nor merely psychological but a fundamental organizing principle of economic growth.

[30] [8], Chapter IV. Fisher formalized the concept of time preference as the fundamental determinant of interest rates, emphasizing that individuals make intertemporal consumption choices based on their preferences for present versus future consumption.

4.5.1 Proof 1: Establishing $R = CGR$

The first proof establishes the identity $R = CGR$, demonstrating that the time preference for consumption is not an externally imposed factor but rather the structural consumption growth rate of the economy. To formally prove that $R = CGR$, we can take the following steps:

Step 1: Start with the fundamental relationships.

Equation (9), the NIE Model derivative:

$$\frac{dY}{dt} = \frac{dC^{\#}}{dt} + \frac{dI^{\#}}{dt}$$

Equation (11), establishing the relationship between R and PRI:

$$R = PRI - \frac{dI^{\#}}{dt}$$

Equation (13), linking PRI and R:

$$PRI - R = \frac{dY}{dt} - \frac{dC^{\#}}{dt}$$

Step 2: Express PRI in terms of R and $dC^{\#}/dt$.

$$PRI = R + \frac{dY}{dt} - \frac{dC^{\#}}{dt}$$

Step 3: Substitute this into the equation defining R.

$$R = \left(R + \frac{dY}{dt} - \frac{dC^{\#}}{dt}\right) - \frac{dI^{\#}}{dt}$$

Step 4: Cancel R on both sides.

$$0 = \frac{dY}{dt} - \frac{dC^{\#}}{dt} - \frac{dI^{\#}}{dt}$$

Step 5: Substitute the NIE Model derivative.

$$0 = \left(\frac{dC^{\#}}{dt} + \frac{dI^{\#}}{dt}\right) - \frac{dC^{\#}}{dt} - \frac{dI^{\#}}{dt}$$

Step 6: Simplify.

$$0 = 0$$

This result demonstrates the validity of the proposition that $R = CGR$ because it confirms that our transformations of the NIE Model derivatives and the relationships between R, PRI, and consumption growth are both logically sound and algebraically consistent. By reaching the universal identity $0 = 0$ without introducing contradictions, we validate that our manipulations correctly isolate R in a way that aligns with the independently derived equation for CGR. Since the final expression for R matches the definition of CGR, it necessarily follows that $R = CGR$.

Step 7: Finally, rewrite Equation (13) to isolate R.

$$R = \frac{dC^{\#}}{dt}$$

Since the CGR equation states,

$$CGR = \frac{dY}{dt} + \frac{dC^*}{dt} = \frac{dC^{\#}}{dt}$$

we conclude that:

$$R = CGR \tag{15}$$

4.5.2 Proof 2: Deriving $d(Y/C)/dt = d(I/C)/dt$

The second proof formalizes the synchronized relationship between income and investment in consumption-normalized terms, showing that:

$$\frac{d(Y/C)}{dt} = \frac{d(I/C)}{dt} \tag{16}$$

Equation (16) demonstrates that changes in income relative to consumption necessarily translate into proportional changes in investment relative to consumption. To prove this, we can proceed as follows:

Step 1: Start with the NIE Model identity:

$$Y = C + I$$

Step 2: Divide both sides by C:

$$\frac{Y}{C} = 1 + \frac{I}{C}$$

Step 3: Differentiate with respect to time:

$$\frac{d}{dt}\left(\frac{Y}{C}\right) = \frac{d}{dt}\left(1 + \frac{I}{C}\right)$$

Step 4: Simplify:

$$\frac{d(Y/C)}{dt} = \frac{d(I/C)}{dt}$$

This result reveals that C functions as a threshold mechanism: when $Y/C > 1$, income flows toward investment; when $Y/C \leq 1$, consumption dominates. The near-perfect empirical correlation ($R^2 = 0.9995$) between $d(Y/C)/dt$ and $d(I/C)/dt$ shown in Figure 12 validates this conclusion.

4.5.3 Proof 3: Connecting R to C as the Threshold Mechanism

Step 1: Start with the fundamental identity. We previously established that:

$$\frac{d(Y/C)}{dt} = \frac{d(I/C)}{dt}$$

This result shows that changes in income relative to consumption evolve in direct proportion to changes in investment relative to consumption.

Step 2: Express $\dfrac{d(Y/C)}{dt}$ *using derivatives.* Applying the quotient rule, we write:

$$\frac{d(Y/C)}{dt} = \frac{\frac{dY}{dt} - R}{C}$$

Given that we have shown that:

$$R = \frac{dC}{dt} = \frac{dC^{\#}}{dt}$$

we substitute this relationship into the equation:

$$\frac{d(Y/C)}{dt} = \frac{\frac{dY}{dt} - \frac{dC^{\#}}{dt}}{C}$$

Step 3: Rearrange the equation. Multiplying both sides by C gives:

$$\frac{d(Y/C)}{dt} \cdot C = \frac{dY}{dt} - \frac{dC^{\#}}{dt}$$

Step 4: Solve for R. Rearranging terms, we obtain:

$$R = \frac{dY}{dt} - \frac{d(Y/C)}{dt} \cdot C$$

Step 5: Interpretation. This equation confirms that R is determined by the difference between the overall growth rate of income, dY/dt, and the rate of change in the income-to-consumption ratio $d(Y/C)/dt$ scaled by C. Since $d(Y/C)/dt = d(I/C)/dt$, we can rewrite the equation as:

$$R = \frac{dY}{dt} - \frac{d(I/C)}{dt} \cdot C$$

Step 6: Conclusion. This result shows that R, the time preference for consumption, is not an independent variable but is directly influenced by changes in income relative to consumption and investment. It reinforces the role of C as the threshold mechanism that determines whether income flows toward consumption or investment–when $d(Y/C)/dt$ is positive (income is growing faster relative to consumption), R decreases, indicating that investment is expanding relative to consumption, and, conversely, when $d(Y/C)/dt$ is negative (income is growing slower relative to consumption), R increases, signaling a reallocation toward consumption. Thus, R is not merely a reflection of time preference but a dynamic regulator of income allocation. This mechanism ensures that consumption and investment remain in balance, reinforcing the argument that time preference is an emergent economic property rather than a static parameter.

4.5.4 Proof 4: The Dynamic Connection Between R, PRI and Investment Growth

The final proof establishes the dynamic connection between R, PRI, and the income-normalized growth rates of consumption and investment. The equation:

$$R = \frac{dC^{\#}}{dt} = \frac{dC^{*}}{dt} + PRI$$

links R to both consumption and investment growth. To establish the relationship between PRI and investment growth, we will show:

$$PRI - R = 100 \cdot \frac{d(I/C)}{dt}$$

This equation clarifies the conditions under which investment expands or contracts.

Step 1: Begin with the definition of PRI:

$$PRI = \frac{dY}{dt}$$

Step 2: Express R in terms of consumption growth:

$$R = \frac{dC^{\#}}{dt}$$

Step 3: Use the relationship between R and PRI:

$$R = \frac{dC^{*}}{dt} + PRI$$

Step 4: Rearranging,

$$PRI - R = -\frac{dC^*}{dt}$$

Step 5: Since we established earlier that:

$$\frac{dC^*}{dt} = -\frac{dI^*}{dt}$$

we obtain:

$$PRI - R = 100 \cdot \frac{d(I/C)}{dt} \tag{17}$$

This result provides a precise mathematical expression for how deviations in PRI drive shifts in investment relative to consumption.

4.5.5 Conclusion

This insight carries profound implications for monetary policy. If R is determined by endogenous business cycle dynamics rather than by external interventions, then central banks cannot sustainably suppress R below CGR without distorting income allocation. This directly contradicts conventional Keynesian and monetarist policy prescriptions, suggesting that artificial rate manipulation is inherently limited in its ability to control investment and consumption patterns.[31]

In contrast to Fisher's view of interest rates as a passive reflection of individual time preference, our framework demonstrates that R functions as a self-adjusting, equilibrium-seeking mechanism that dynamically regulates the economy. The fact that R aligns with empirical business cycle trends underscores the predictive power of this model. By establishing that business cycles are an emergent consequence of economic structure rather than an anomaly requiring correction, the Augmented NIE Model Business Cycle offers a new paradigm for understanding economic fluctuations. This reformulation advances Fisher's foundational work by integrating time preference with real-world cyclical dynamics, offering a model in which interest rates, growth, and investment behavior are

[31] See, for example, [17] for a discussion of Keynesian policy prescriptions, and [3] for a critique of the political implications of Keynesian economics.

not separate variables but interconnected components of a unified system.[32]

5 The Impossibility of a Steady-State Equilibrium

Our central premise is that the economy cannot sustain an equilibrium where consumption and investment grow at the constant growth rate (CGR) indefinitely. Because the ultimate purpose of economic activity is consumption, agents will not indefinitely forgo consumption in favor of the investment required for continued growth. This inherent tension between the desire for present consumption and the need for investment to sustain growth creates a dynamic instability that prevents the economy from settling into a steady state.

A necessary condition for a steady-state continually progressing economy (CPE) is that consumption (C) and investment (I) grow at the same rate as income (Y), ensuring that their relative shares of income (C/Y and I/Y) remain constant. Mathematically, this requires:

$$\frac{dC}{dt} = \alpha \frac{dY}{dt}, \quad \frac{dI}{dt} = (1-\alpha)\frac{dY}{dt}$$

for some fixed α, which represents the fixed proportion of income growth allocated to consumption. The PRI-R Differential ($PRI - R$) cannot persist at any fixed value in a *CPE*, because it always leads to one of three outcomes:

1. $PRI - R > 0$. A positive PRI-R differential signals arbitrage opportunities for further growth. In an efficient market, firms will pursue additional investment until the differential is eliminated. This leads to an increase in investment relative to consumption, altering the income allocation.

2. $PRI - R < 0$. A negative PRI-R Differential implies that investment is yielding less than the cost of capital. This forces disinvestment, reducing income and triggering an adjustment in consumption and investment shares.

3. $PRI - R = 0$. When the PRI-R Differential equals zero, no further investment can be justified. This results in stagnant income growth, preventing the economy from progressing at the CGR.

[32] See [8] for Fisher's view on interest rates and time preference, and [6] and [7] for additional context.

Thus, any particular PRI-R Differential is inherently self-correcting, making it impossible to maintain a stable, proportional relationship between consumption, investment, and income over time. The allocation of income between consumption and investment must continuously adjust in response to these dynamics, preventing a steady-state equilibrium in the *CPE*.

Even in a zero-growth scenario, where income, consumption, and investment stagnate, the tension between consumption and investment preferences will disrupt the steady state. Stability in a zero-growth scenario requires the PRI-R Differential to be zero, because any other rate would mean that the demand for marginal consumption (MC) is unsatisfied. However, in a functioning economy, the pure interest rate must be greater than zero because it reflects human time preference, and the oscillation between consumption and investment would manifest.

Thus, the Augmented NIE Model inherently describes an economy in perpetual disequilibrium, where consumption and investment shares continually adjust in response to internal dynamics.

The fact that an equilibrium where income, consumption, and investment grow steadily at the CGR is not theoretically possible does not rule out the possibility of sustained growth in all NIE Model Variables at levels sufficient to support secular growth at the CGR. We will now turn to the demonstration that the latter requires a stable disequilibrium and an explanation of how it can occur under the dynamic interplay between consumption and investment preferences, mediated by the pure interest rate (R) and fueled by the potential return on investment (PRI).

6 The Cyclical Phases of Investment and Consumption: Preconditions, Dynamics, and Outcomes

The potential for sustainable secular (long-term) growth rides on the existence of a corresponding excess of potential return on investment (PRI) over the cost of borrowing (R) across, but not within, business cycles. When the PRI-R Differential ($PRI - R$) is greater than zero, investors are incentivized to defer consumption and to borrow to make profitable investments, promoting economic growth. To the extent that the PRI-R Differential becomes negative, economic agents will consume an increasing share of income, causing the investment rate ($dI^{\#}/dt$) and the amount of funds available for investment to decline. These forces do not tend to produce a static equilibrium state but instead interact dynamically to create a constantly oscillating system that results in the overall long-term growth of income at the CGR, with the rate of change of income (dY/dt) oscillating around the CGR, as demonstrated in Section 4.3.2.

To understand the business cycle dynamics, we posit a continually progressing economy (CPE) as a heuristic device. Our CPE is a simplified model of an economy where income grows at a long-term average rate, which we represent as a linear CGR. By referencing the CGR as a baseline, we can identify periods of above- and below-trend growth, as income growth (dY/dt) oscillates around it. We can also delineate phases of the business cycle by examining the relative rates of change in income-normalized consumption (dC^*/dt) and income-normalized investment (dI^*/dt), as they oscillate above and below zero. The horizontal line at zero is referred to as the "Zero Line", which when analyzing variables that have been normalized for income, represents dY/dt as a static reference. We income-normalize these variables to focus on how the share of income allocated to consumption and investment shifts. The income-normalization process allows us to isolate the dynamic interplay of investment and consumption as it drives cycles using the continuous measures provided by calculus, providing the methodology for testing our theory.

The business cycle comprises two primary phases: the Investment Phase and the Consumption Phase. The Investment Phase occurs when the income-normalized rate of change in investment (dI^*/dt) is positive, signifying that the economy is allocating an increasing share of income to investment in projects whose potential return on investment (PRI) exceeds the time preference for consumption (R). Borrowing plays a critical role in amplifying the Investment Phase; by simultaneously deferring consumption and borrowing to fund investment, households and firms enable an increase in investment and an associated increase in income growth, allowing the economy to exceed the CGR.

During the Investment Phase, the direct relationship between dI^*/dt and dY/dt shown in Figure 6 (*Y/Y Changes in Real GDP and Normalized Investment*) and Figure 12 (*Correlation of Income and Investment Normalized for Consumption*) creates a positive feedback loop. As investment accelerates, the economy increasingly allocates income to projects, utilizing available resources to their fullest extent, which drives up productivity and income growth. This process temporarily increases the aggregate potential return on investment (PRI) as new opportunities are developed and existing investments yield higher returns due to economies of scale or network effects. For a time, the rate of increase in PRI outpaces that of the pure interest rate (R), widening the PRI-R Differential. This widening reflects not only the rising aggregate returns from investment but also the rapid pace at which new projects are being initiated.

This self-reinforcing mechanism operating during the Investment Phase continues until dI^*/dt and $dI^{\#}/dt$ approach their peak at the Central Growth Rate (CGR), a point at which income-normalized consumption is at its trough due to the Zero-Sum Dynamic ($dC^*/dt = -dI^*/dt$). At the peak of the Investment Phase, income growth is maximally allocated to investment, corresponding to the maximum deviation of dI^*/dt from the Zero Line and dY/dt from the CGR. These peaks can be expressed as:

$$\frac{dI^*}{dt_{\text{peak}}} = CGR$$

and

$$\frac{dY}{dt_{\text{peak}}} = CGR + \frac{dI^*}{dt} = 2 \cdot CGR$$

The trough of income-normalized consumption growth (dC^*/dt) always occurs at the same point as the peak of income-normalized investment growth (dI^*/dt) within each oscillatory cycle. This relationship arises directly from the Zero-Sum Dynamic ($dC^*/dt = -dI^*/dt$), and it reflects the underlying oscillation of income growth (dY/dt) around the CGR as it cycles between its maximum ($2 \cdot CGR$) and minimum (0) values. As we will demonstrate in Section 7.7.3 (*The Augmented NIE Variable Relationships*), we can use this relationship to mathematically derive that the rate of change in income-normalized consumption at its trough is always equal to the inverse of the Central Growth Rate, such that:

$$\frac{dC^*}{dt}_{\text{trough}} = -CGR.$$

The alignment of (dC^*/dt_{trough}) with ($-CGR$) signifies that a maximum deferral of consumption has occurred, supporting the maximum allocation of income growth to investment during the Investment Phase.

From the peak of the Investment Phase, the positive feedback mechanism begins to unwind and turn negative. As the potential return on investment (PRI) declines and income growth (dY/dt) slows, investment demand (dI/dt) diminishes, and all three fall relative to the time preference for consumption (R), which remains supported by the above-trend growth of the early business cycle and the costs of replenishing or allowing the depletion of the resources that yielded that growth.[33] The declines in dI^*/dt and dY/dt feed on each other, reinforcing the deceleration. The PRI-R Differential narrows and turns negative and, along with it, the income-share adjusted ($dI^\#/dt$) and income-normalized rate of change in investment (dI^*/dt). Borrowing, which previously fueled the acceleration of investment, becomes less justifiable as the marginal returns on investment converge with the marginal costs of deferred consumption. The income-normalized rate of change in consumption (dC^*/dt) begins to rise. This increase in consumption relative to income occurs while the income-share adjusted rate of consumption ($dC^\#/dt$) continues to track the CGR, demonstrating that the dominance of consumption is occurring due to the decline in income growth, investment, and their associated rates of change. The decline in investment and simultaneous relative increase in consumption signifies the impending transition to the Consumption Phase in which a new consumption-driven dynamic will dominate economic activity. The Consumption Phase begins when the Investment Phase concludes, marked by the marginal propensity to invest (MI), income-share adjusted investment growth ($dI^\#/dt$), and the PRI-R Differential all approaching zero.

The active reallocation of resources during the Consumption Phase includes enjoyment of the above-trend growth from the Investment Phase, and natural depreciation of capital as well as consumption akin to Schumpeter's "creative destruction"[34], a process by which the economy begins phasing out outdated investments and using the depreciated value of fixed capital to fund new investment directly.[35] The Consumption Phase peaks when the consumption growth rate relative

[33] See [31] and [32]

[34] [31]

[35] As will be highlighted in Section 7.1, in our model, we make a unique adjustment by adding consumption of fixed capital to Personal Consumption Expenditures (PCE), creating what we call Gross Personal Consumption Expenditures (GPCE). This adjustment accounts for the consumption of capital through depreciation, recognizing the economic activity involved in replacing or maintaining capital goods, and allows us to use Net Domestic Private Investment (NDPI) as a proxy for income-producing investment.

to income is at its maximum. The peak occurs when the marginal propensity to consume (MC) is at its highest, which coincides with the trough of (dI^*/dt). At the peak of (dC^*/dt):

$$\frac{dC^*}{dt}_{\text{peak}} \approx \frac{dC^*}{dt} + \frac{dY}{dt}$$

and

$$\frac{dY}{dt_{\text{trough}}} = CGR - \frac{dC^*}{dt} = 0$$

Equation (14) demonstrates that the peak of dC^*/dt, when $dY/dt = 0$, is equal to the Central Growth Rate (CGR). This result follows because, by symmetry, the peak of dC^*/dt must always correspond to the trough of dI^*/dt, which, in our model, always corresponds to $-CGR$. [36]

Although the economy slows during the Consumption Phase, this deceleration does not hinder the ongoing progress of the foundational inputs, such as population growth, resource discoveries, and technological advancements, sustaining a *CPE*. Instead, the components driving a regenerating PRI—including increased marginal labor productivity, reduced input costs (e.g., wages), and technological innovation—remain in continuous operation, which improves economic efficiency and profitability, laying the groundwork for the next Investment Phase. As these factors gradually create increases in potential returns relative to the time preference for consumption, and therefore, as PRI surpasses R and the PRI-R Differential becomes positive, a renewed demand for investment begins to develop, setting the stage for the next cycle of economic growth.

Economists have drawn parallels between economic fluctuations and the oscillatory patterns seen in predator-prey models, which bear a resemblance to the investment-consumption dynamic, albeit with important distinctions such as nonlinear dynamics and lagged feedback loops.[37] The Augmented NIE Model Business Cycle demonstrates that, like predator-prey systems that resist a fixed equilibrium, a simple, stable equilibrium is impossible in a *CPE*. Instead, our system operates in perpetual disequilibrium, centered around the CGR and the Zero Line and driven by the competing demands of consumption and investment. However, unlike the predator-prey systems

[36] See Section 7.7 (*Building the Augmented NIE Model Business Cycle*) for the derivation of these equations.
[37] [12], [28],[23] and [34].

which depend on lagged responses, the Augmented NIE Model Business Cycle features smooth and mechanical transitions based on linear income allocation. As demonstrated in Section 7.4, the curves in our system include saw-toothed and sinusoidal components, and are non-parabolic and exponential with no lag in the Zero-Sum Dynamic. [38]

An interesting and important question for future research, which is beyond the scope of this paper, is whether the borrowing that occurs during the Investment Phase, by temporarily exceeding the Central Growth Rate, results in a higher long-term CGR compared to an identical economy where borrowing is restricted. Such borrowing could raise the secular growth rate by advancing future production and changing the size and scope of feasible projects. Investments in innovative technology, major infrastructure, or large-scale industrial projects that are not possible or profitable on a pay-as-you-go basis may permanently boost the economy's productive capacity.

Our heuristic Augmented NIE Model Business Cycle of a CPE with a constant CGR offers a simplified yet robust framework for understanding the dynamic interplay between investment and consumption. While this model assumes a steady CGR, real-world economies grow with interruptions, such as recessions and expansions, introducing variability into the CGR within and across business cycles. These fluctuations cause the peaks and troughs of the investment-consumption cycle to shift in response to changing growth rates, external shocks, and structural adjustments; nevertheless, the Zero-Sum Dynamic ($dC^*/dt = -dI^*/dt$), driven by the PRI-R Differential ($PRI - R$), continues to govern the reallocation of income and the oscillatory nature of economic cycles. We will extend the insights of the Augmented NIE Model Business Cycle to economies with varying CGRs, demonstrating that the oscillatory dynamics driven by these core factors remain robust, offering valuable tools for understanding both steady and interrupted growth trajectories, and showing how the model remains relevant with real-world conditions. The mathematical and empirical demonstration of these claims is the subject of Section 7 (*Evaluating the Theory*).

[38] While insightful, predator-prey models (e.g., Lotka-Volterra) often fail to reach stable equilibrium due to inherent nonlinearity, sensitivity to parameter variations, and time lags between prey and predator populations, leading to cyclical behavior instead. Real-world complexities such as alternative food sources and environmental changes further destabilize potential equilibria, resulting in dynamic and complex outcomes. For a detailed mathematical explanation, see [23]; [34] 2 (1926): 31-113.

7 Evaluating the Theory

We conducted various tests and analyses to demonstrate the theory's explanatory power. Our inquiry specifically aimed to demonstrate:

1. the efficacy of the GDP Proxy in accurately modeling economic activity;
2. that the ultimate purpose of economic activity is consumption, and for sustainable economic growth at a positive CGR, consumption and investment must grow harmoniously with income;
3. that changes in the rates of consumption and investment fully account for the existence and persistence of the Business Cycle;
4. that the formal shape and persistence of cyclical fluctuations in the rates of change of consumption and investment conclusively support our assertion of non-decaying, secular cycles in a CGR economy;
5. that changes in the rates of consumption and income significantly explain variations in interest rates, accurately reflecting Time Preference and the cost of borrowing;
6. the construction of a credible Augmented NIE Model Business Cycle using only time series data representing the NIE Model Variables (Y, C, I) and deriving R and PRI directly from them;
7. that under the Augmented NIE Model Business Cycle, the economy efficiently realizes the potential return on investment (PRI) for a given time preference (R);
8. that consumption growth robustly tracks the Central Growth Rate (CGR), while investment growth dynamically adjusts to optimally realize potential output.

Below, we present our findings and provide supporting data and analysis. It is important to note that while we sometimes employ Augmented NIE Variables normalized for income to remove fluctuations caused by income changes, our findings decisively demonstrate the broader applicability of the model to the aggregate dynamics of income, consumption, and investment in the economy, as shown in Figure 21 (*Augmented NIE Model Business Cycle Based On Real-World Data*) and the accompanying discussion in 7.7.4.

7.1 The Augmented NIE Model, GPCE, NDPI, and the GDP Proxy

Our analysis employs proxies for the variables in the National Income Equilibrium (NIE) Model. The core model simplifies the economy to the foundational NIE Model Equation 1: $Y = C + I$, where Y represents aggregate income, and C and I are aggregate consumption and investment, respectively. This approach intentionally excludes government spending and net exports to focus on the core dynamics of private consumption and investment as the principal drivers of the business cycle. To operationalize this framework, we select appropriate real-world data series representing these concepts.

We use Personal Consumption Expenditures (PCE) as the primary component of consumption, reflecting spending by individuals on goods and services. Similarly, we use Gross Private Domestic Investment (GDPI) as the primary component of investment, encompassing business fixed and residential investments. However, GDPI also includes Consumption of Fixed Capital (CFC), which we categorize as a consumption expenditure rather than an addition to productive capacity. To address this, we use Net Domestic Private Investment (NDPI), calculated by the Bureau of Economic Analysis (BEA) as GDPI minus CFC. This adjustment ensures that NDPI represents only the portion of investment that contributes to productive capacity, excluding the replacement of depreciated capital.

To comprehensively capture all forms of private consumption, we add CFC to PCE, creating Gross Private Consumption Expenditure (GPCE), which includes the consumption of fixed capital and personal spending. Finally, to derive our GDP Proxy, we sum GPCE and NDPI, explicitly modeling all private economic activity as either consumption or investment. These adjustments ensure that our empirical measures align directly with the theoretical framework of the NIE Model, providing an undiluted representation of its internal relations and dynamics.

Throughout this Section 7, we will refer to Gross Consumption (GC) interchangeably with consumption (C), using the former to emphasize its inclusion of Consumption of Fixed Capital (CFC) and the latter to conform to the variables in our formal Augmented NIE Model Equations. The same approach applies to Net Private Domestic Investment (NPDI) and investment (I).

Equation (1) ($Y = C+I$) differs from the BEA's Expenditure Approach to GDP ($GDP = C+I+G+NX$, where G is government spending and NX is net exports)by isolating private consumption and investment. The Expenditure Approach incorporates additional components beyond Personal Consumption Expenditures (PCE), Net Domestic Private Investment (NDPI), and Consumption of Fixed Capital (CFC), which are the only data series we use to construct our GDP Proxy.

Our first key finding demonstrates that our GDP Proxy captures approximately 86% of actual, nominal GDP, with a correlation of 0.9995 over the study period from 1962 Q1 to 2024 Q3 (Full Study Period). This remarkable correlation underscores the validity of our analytical framework and confirms the integrity of using GPCE and NDPI as proxies for consumption (C) and investment (I).

Figure 1 (*Correlation of* GDP Proxy (GPCE *plus* NDPI) *to* GDP) presents a scatter plot comparing our GDP Proxy to actual GDP.

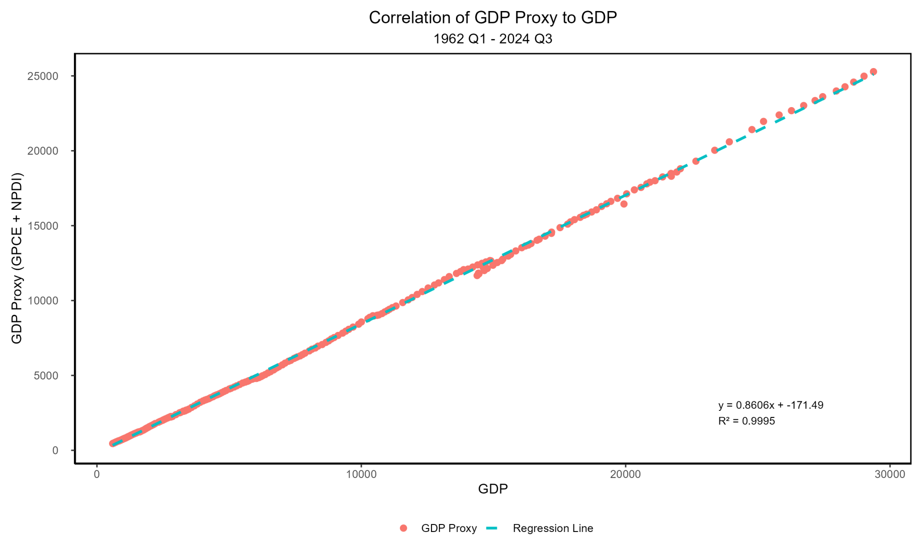

Figure 1: *Correlation of GDP Proxy (GPCE plus NDPI) to GDP.* This chart demonstrates that the sum of GPCE and NDPI accounts for virtually all fluctuations in GDP. It highlights the efficacy of using GPCE, NDPI, and their sum (GDP Proxy) to validate the Augmented NIE Model Business Cycle and its derived variables.

7.2 The Relationship Among Consumption, Investment, and Income

Figure 2 (*Y/Y Changes in Investment, Consumption, and GDP*) shows the year-over-year rates of change of the three raw NIE Model Variables (GPCE, NDPI, and their sum, the GDP Proxy) over the Full Study Period. We calculated the derivatives of GPCE and NDPI without adjusting their relative income shares. Because these series have different base levels, their absolute rates of change are not directly comparable. As anticipated and explained in the text accompanying Equation (9)

($dY/dt = dC^{\#}/dt + dI^{\#}/dt$), the unadjusted derivatives do not sum to the income derivative. This lack of comparability highlights a fundamental issue: comparing changes in consumption and investment without accounting for their income shares fails to accurately reflect the allocation of economic activity and instead only indicates the raw percentage changes of those two variables. Figure 2 demonstrates the limitations of using unadjusted rates of change.

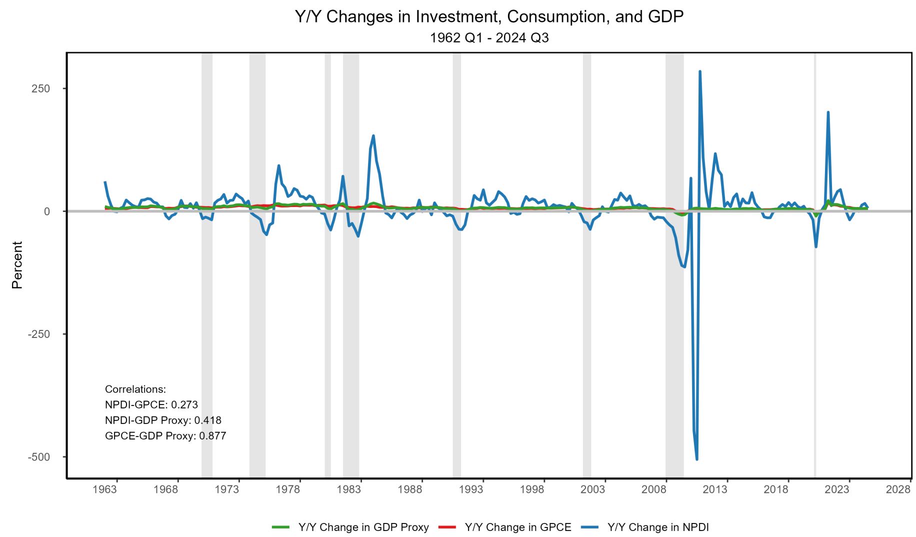

Figure 2: *Y/Y Changes in Investment, Consumption, and GDP:* This chart illustrates that because nominal consumption and investment have different base levels, merely comparing their respective rates of change without any income-share adjustment yields little information about their relationships. Notably, the correlation between GPCE and NDPI is positive ($R^2 = 0.273$), in contrast to the income-normalized case, where the Zero-Sum Dynamic is revealed. (See Figure 4(*Changes in Normalized Consumption and Investment*).)

Although Equation (7) ($dY/dt = dC/dt + dI/dt$), a definitional derivation from basic calculus, does not hold mathematically when calculated with respect to annual percent changes of unadjusted data series, it helps evaluate our central principle: all three NIE Model Variables must grow at similar rates to sustain long-term growth. To assess this principle, we calculated the average percent changes and integrals (representing cumulative totals of the underlying series) using the trapezoidal method for each NIE Model Variable. The trapezoidal method multiplies the average rate of change between consecutive quarters by 0.25 (representing the quarter-year interval) and sums the results over the period. We performed these calculations for the Full Study Period and the study subperiod from 1969 Q4 to 2020 Q1 (Study Subperiod), which spans the eight recessions within the Full Study Period.

Table 1 (*Average Changes and Integrals of the Raw NIE Variables*) shows the NIE Model Variables' average year-over-year percent changes and integrals across both periods. The average percent changes in the NIE Model Variables are closely aligned, particularly in the Study Subperiod. Similarly, the integrals of the NIE Model Variables align closely in both periods, except for the NDPI integral in the Study Subperiod. We attribute this deviation to the extreme variability of NDPI during events like the Great Recession, which saw substantial capital consumption and a decline in investment exceeding 500%. These results provide sufficient evidence that our core principles hold when using the raw data series: economic activity is primarily consumption-driven, and sustained income growth requires long-term proportional (but not necessarily simultaneous) growth in consumption, investment, and income.

Table 1: *Average Changes and Integrals of the Raw NIE Variables*

1962 Q1 – 2024 Q2 (Full Study Period)		
Variable	**Average Y/Y Percent Change**	**Integral**
GDP Proxy	6.76	422.03
GPCE	6.79	424.61
NDPI	7.10	437.32
1969 Q4 – 2020 Q1 (Study Subperiod)		
Variable	**Average Y/Y Percent Change**	**Integral**
GDP Proxy	6.58	332.98
GPCE	6.68	337.86
NDPI	6.55	284.73

Table 1 demonstrates that the secular growth in the raw NIE Model Variables is nearly equal, reinforcing that long-term proportional growth in consumption, investment, and income is essential for sustained economic expansion.

To accurately reflect the contributions of consumption and investment to income growth, and to satisfy Equation (9) ($dY/dt = dC^{\#}/dt + dI^{\#}/dt$), we income-share adjusted GPCE and NDPI by their respective shares of the GDP Proxy. Figure 3 (*Y/Y Changes in GDP, Consumption, and Investment*) illustrates the income-share adjusted NIE Model Variables over the Full Study Period, showing how they align with the business cycle.

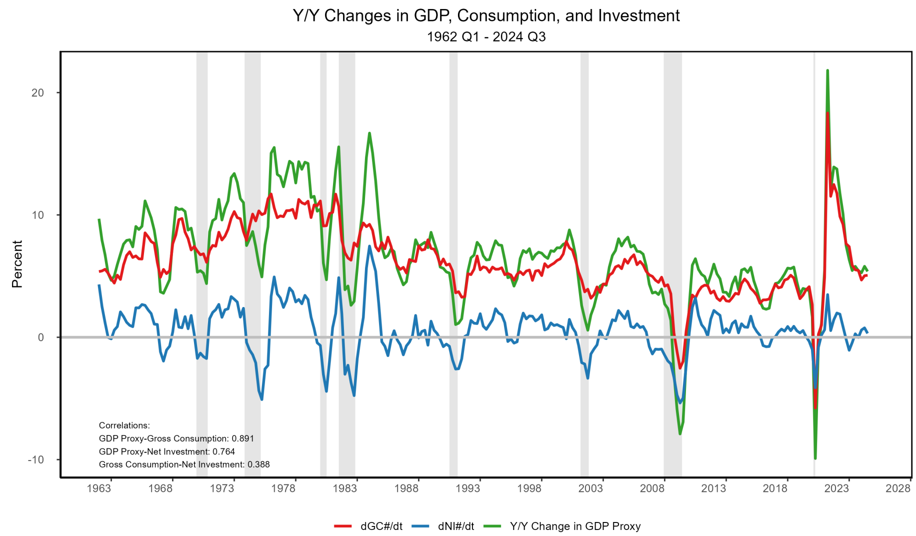

Figure 3: *Y/Y Changes in GDP, Consumption, and Investment:* This chart shows that income-share adjusted changes in consumption, investment, and income generally move in the same direction, even though income-normalized changes in consumption and investment are inversely correlated. See Figure 4 (*Changes in Normalized Consumption and Investment*).

To verify that Equation (9) ($dY/dt = dC^{\#}/dt + dI^{\#}/dt$) holds mathematically for the income-share adjusted NIE Model Variables during the Full Study Period, we calculated their average percent changes: GDP Proxy, 6.76; GPCE, 6.23; and NDPI, 0.53, which sum exactly. Because the GDP Proxy is defined as the sum of GPCE and NDPI, these results confirm the internal consistency of our analytical structure rather than directly validating the economic theory.[39]

7.3 Stable Disequilibrium and the Business Cycle:

Equation (10) ($dC^*/dt = -dI^*/dt$) is perhaps the most crucial equation in our analysis because, when applied to real-world consumption and investment data, it demonstrates that income-normalized consumption and income-normalized investment oscillate in a stable, cyclical disequilibrium pattern. Equation (10) demonstrates the Zero-Sum Dynamic, where any increase in the share of income devoted to one category directly decreases the share allocated to the other. The Zero-Sum Dynamic ensures that resources are continuously reallocated between the consumption

[39] The integrals of the income-share adjusted NIE Model Variables showed an anomaly similar to the raw data series, confirming the limitations of the trapezoidal method when applied to high-variance data series.

and investment, determining the manner in which income is pulled forward and consumed within each cycle. Figure 4 (*Changes in Normalized Consumption and Investment*) depicts this perfect inverse relationship[40] between these two variable derivatives (R = -1) throughout the Full Study Period, which occurs because the normalization process forces the two variables to sum to a constant. As their rates of change represent the derivative of this constant sum, their relationship must be perfectly reciprocal.

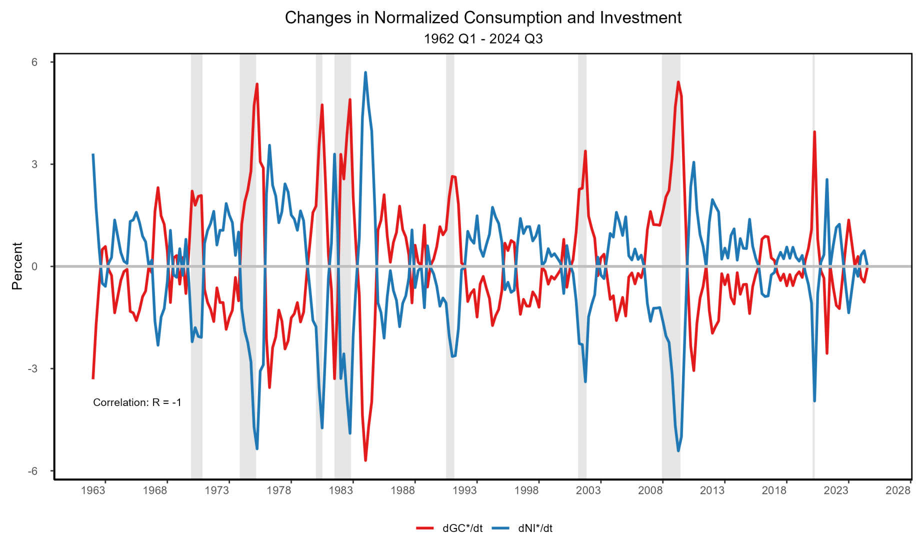

Figure 4: *Changes in Normalized Consumption and Investment:* This chart shows that changes in consumption and investment, when normalized for income, are equal and opposite (180 degrees out of phase) and persist across business cycles. Notably, the peaks in (dGC^*/dt) and the troughs in (dNI^*/dt) align with recessionary periods, supporting our analysis in Section 6.

To confirm the visual immediacy of the relationship between the derivatives of income-normalized gross consumption (dGC^*/dt) and income-normalized net investment (dNI^*/dt), we conducted the cross-correlation analysis shown in Figure 5 (*Cross-Correlation between* GC^* *and* NI^*). The results confirm the absence of any significant lag, indicating the intimate real-time linkage of changes in consumption and investment. The linkage between GC^* and NI^* means that the rate of change in consumption must always occur contemporaneously with, but in the opposite direction to, changes in the rate of change of investment, as implied by Equation (10) ($dC^*/dt = -dI^*/dt$), demonstrating

[40]The correlation coefficient ($R = -1$) between dGC^*/dt and dNI^*/dt is an expected result of our income normalization process, robustly validating our normalization and income-sharing methodology.

how consumption serves as a mediating mechanism for optimizing investment.

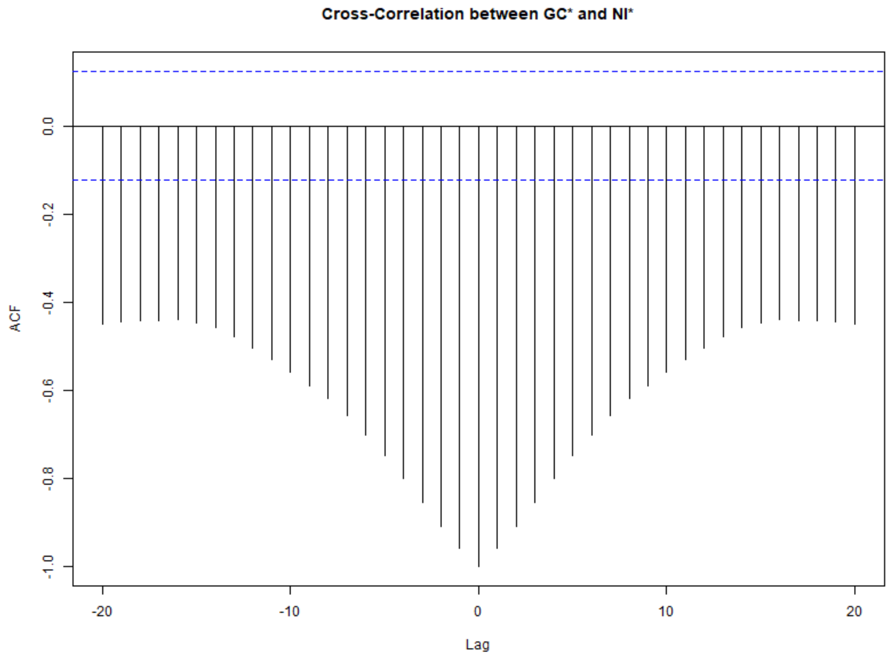

Figure 5: *Cross-Correlation between* GC^* *and* NI^**:* This chart shows that the rates of change in gross consumption and net investment are equal but opposite and occur contemporaneously, confirming that the economy optimizes investment through the mediation of consumption. See also Figure 12 (*Correlation of Income and Investment Normalized for Consumption*).

Although the preceding analysis demonstrates the alternating dominance of investment and consumption in driving the business cycle, it leaves the question of whether this phenomenon alone can explain the expansions and contractions that define it. To address this, we examined the critical interaction between income-normalized investment (dNI^*/dt) and real income growth, represented by our real GDP Proxy minus the GDP deflator, to determine whether declines in investment alone may be sufficient to trigger contractions in income, as shown in Figure 6 (*Y/Y Changes in Real GDP and Normalized Investment*).

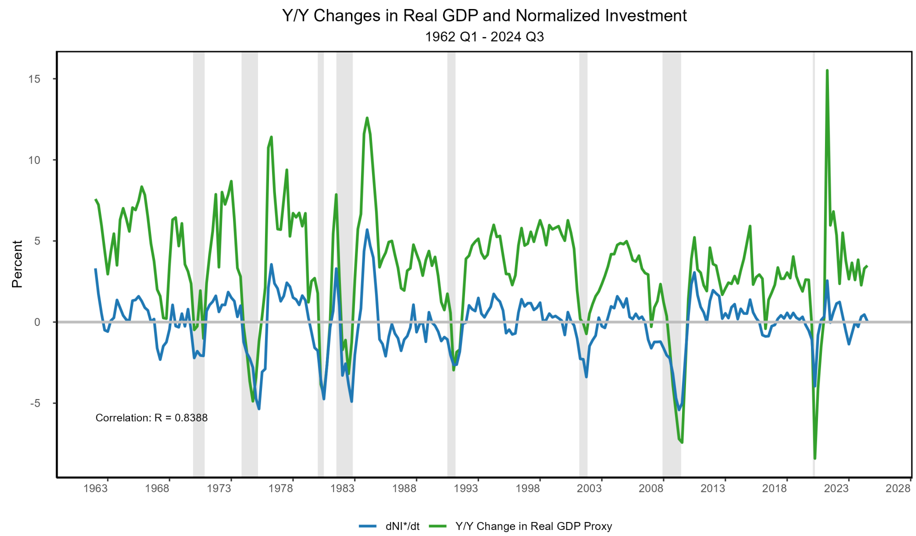

Figure 6: *Y/Y Changes in Real GDP and Normalized Investment:* This chart demonstrates that declines in (dNI^*/dt) below the Zero Line, indicating that income-normalized investment is insufficient to sustain income growth at its current rate, consistently coincide with or precede recessionary periods.

Figure 6 shows that changes in income-normalized net investment (dNI^*/dt) and real GDP exhibit synchronized peaks, troughs, and shifts at critical points in each business cycle, with declines in (dNI^*/dt) below the Zero Line occurring at or near the beginning of recessionary periods. This supports the conclusion that reductions in investment alone may be sufficient to explain contractions in income.

Analysis of the relationship between changes in income-normalized consumption (dGC^*/dt) and real income, shown in Figure 7 (*Y/Y Changes in Real GDP and Normalized Consumption*), demonstrates that consumption's share of income increases at or near the beginning of recessionary periods. This fulfills one of the two recession conditions described in Section 4.4, that recessions occur when $dC^*/dt >= 0$.

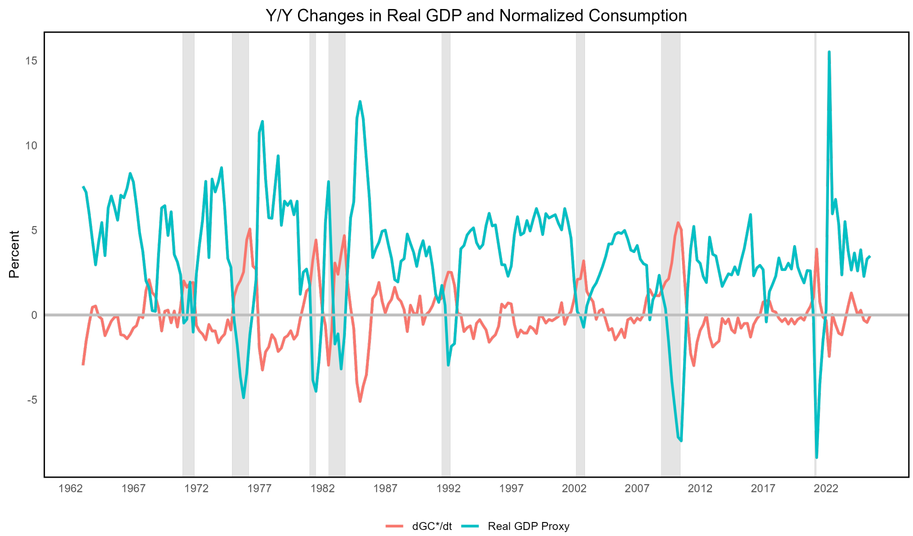

Figure 7: *Y/Y Changes in Real GDP and Normalized Consumption:* This chart shows that increases in dGC^*/dt above the Zero Line, indicating changes in income-normalized consumption are depriving the economy of the investment needed to sustain income growth at its current rate, consistently coincide with or precede recessionary periods.

While Figure 6 and Figure 7 demonstrate that declines in income-normalized investment ($dI^*/dt < -CGR$) and rises in income-normalized consumption ($dC^*/dt > CGR$) coincide with recessions, their marginally smaller oscillatory amplitudes compared to dY/dt may not be viewed as dispositive on the question of whether income-normalized variables fully explain income fluctuations. Figure 29 (*Excess of Y/Y Change in GDP over Trend vs Y/Y Change in Normalized Investment*), which directly compares the amplitudes of dNI^*/dt and the year-over-year change in the GDP Proxy, provides compelling evidence that oscillations in income-normalized net investment are sufficiently large to account for fluctuations in income.

7.4 Form of Changes in Consumption and Investment: Correlations Between Derivatives

Having concluded that the evidence supports the power of changes in the rates of consumption and investment as explanatory of cyclical economic activity, we turn our attention to an examination of the form and persistence of those oscillations. We begin with examining the nature of changes in income-normalized consumption (C^*) and income-normalized investment (I^*) by evaluating correlations among their first, second, third, and fourth derivatives. These correlations follow a specific pattern: high correlations between successive derivatives (e.g., dC^*/dt and d^2C^*/dt^2, d^2C^*/dt^2 and d^3C^*/dt^3, d^3C^*/dt^3 and d^4C^*/dt^4), moderate correlations between dC^*/dt and d^3C^*/dt^3 and between d^2C^*/dt^2 and d^4C^*/dt^4, and a low correlation between dC^*/dt and d^4C^*/dt^4. We intentionally examined this sequence to establish the overall shape of the oscillations, which we theorize exhibit characteristics of sinusoidal and sawtooth components. While sinusoidal features reflect the smooth and periodic nature of the oscillations, the sawtooth elements capture sharper transitions or asymmetries in the movement of the underlying series. stable and persistent oscillatory behavior across business cycles, as demonstrated in Figure 8 (*First, Second, Third, and Fourth Derivatives of Normalized Consumption*), which shows the consistent and cyclical nature of the rate of change in income-normalized consumption, contrasting with parabolic or decaying patterns. The high correlations between successive derivatives—first and second, second and third, and third and fourth—indicate a strong relationship between changes in the rate of growth and the concavity and "jerk" of the curve. These characteristics are typical of sinusoidal and similar periodic functions, suggesting a smooth, continuous, and stable oscillatory movement in the underlying series. The analysis reveals a smooth and predictable overall trend, characteristic of stable oscillatory behavior. Nonetheless, the observed weakening of correlations between earlier and higher-order derivatives suggests that later changes in consumption patterns introduce increasing complexity. Rather than indicating a dissipation of oscillatory behavior, this complexity reflects a persistent structural evolution, where changes continue to manifest in a non-random yet increasingly intricate manner. The stability of the oscillations underscores their persistent consumption dynamics, a topic discussed in Section 6 (*The Cyclical Phases of Investment and Consumption: Preconditions, Dynamics, and Outcomes*) and which we will further explore in Section 7.7.1 (*Theoretical Foundations for the Augmented NIE Model Business Cycle*).

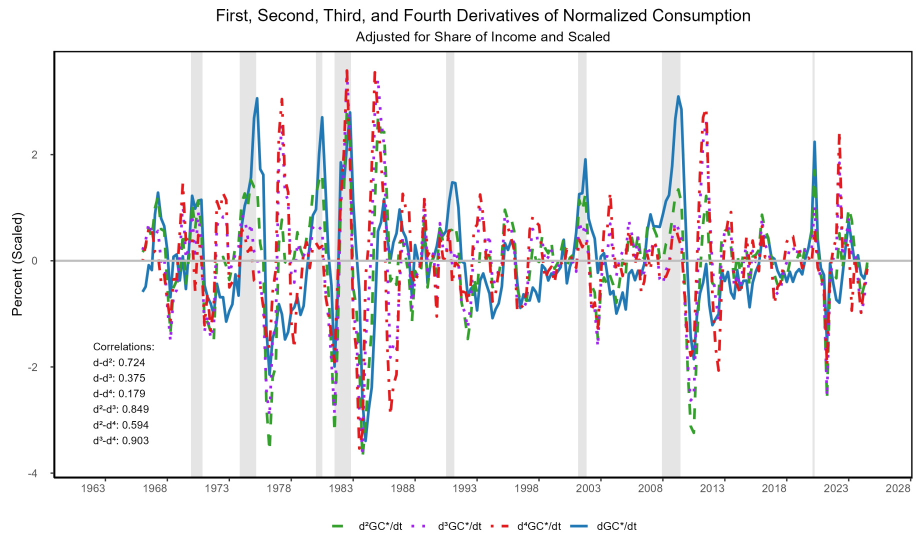

Figure 8: *First, Second, Third, and Fourth Derivatives of Normalized Consumption*: This chart demonstrates that the rate of change in income-normalized consumption is stable and exponential rather than parabolic or decaying. The consistent stability and absence of decay indicate the persistent nature of oscillations across all business cycles.

Likewise, Figure 9 (*Rolling Standard Deviations of Normalized Consumption Derivatives*) depicts the rolling standard deviations of these derivatives, revealing increasing variability at higher orders. This pattern suggests that while the overall trajectory of consumption growth remains stable, higher-order derivatives intensify the effects of economic fluctuations. The observed amplification reflects a systematic, oscillatory pattern that persists over time, reinforcing the enduring nature of these dynamics.

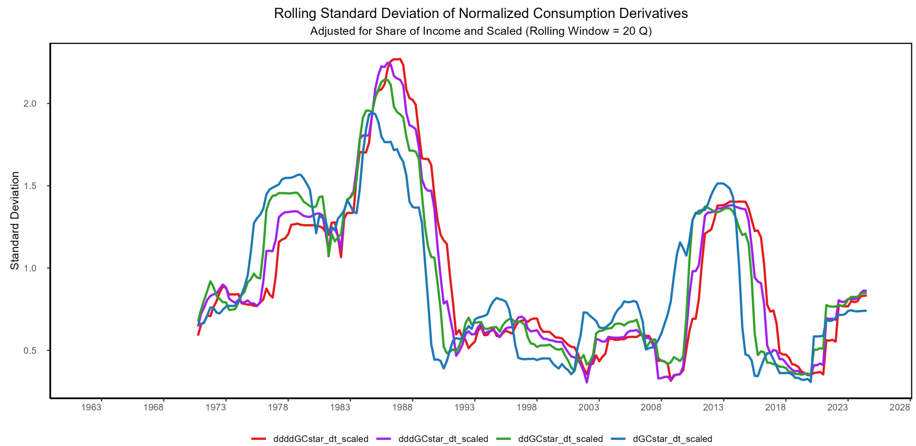

Figure 9: *Rolling Standard Deviations of Normalized Consumption Derivatives*: This chart illustrates the increasing variability with each higher-order derivative. The observed pattern reflects that cyclical changes in consumption are persistent, systematic, and non-decaying.

The changes in income-normalized investment derivatives exhibit a similar pattern of stability and persistence to that of consumption. As shown in Figure 10 (*First, Second, Third, and Fourth Derivatives of Normalized Investment*), the rate of change in income-normalized investment is also stable and consistent, rather than exhibiting parabolic or decaying patterns, confirming that oscillatory behaviors extend to investment dynamics. The high correlations between successive derivatives reinforce the smooth, systematic nature of these changes, while the diminishing correlations between lower- and higher-order derivatives suggest increasing independence of higher-order effects, consistent with our findings for income-normalized consumption. Likewise, Figure 11 (*Rolling Standard Deviations of Normalized Investment Derivatives*) depicts the rolling standard deviations of these derivatives, revealing increasing variability at higher orders. This pattern suggests that while the overall trajectory of investment growth remains stable, higher-order derivatives intensify the effects of economic fluctuations. The observed amplification reflects a systematic, oscillatory pattern that persists over time, reinforcing the enduring nature of these dynamics.

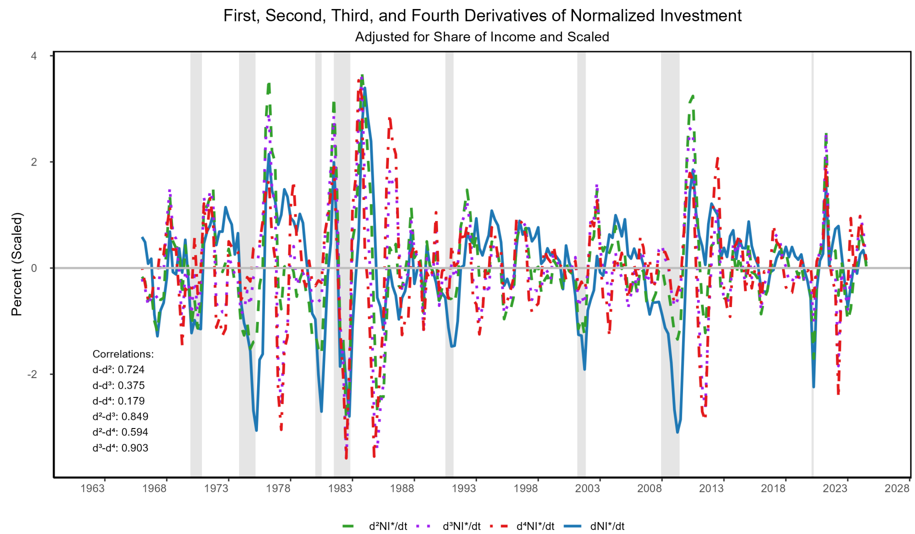

Figure 10: *First, Second, Third, and Fourth Derivatives of Normalized Investment*: This chart demonstrates that the rate of change in income-normalized investment is stable and consistent, rather than parabolic or decaying. The close parallels with the consumption series confirm the persistent nature of oscillations across business cycles.

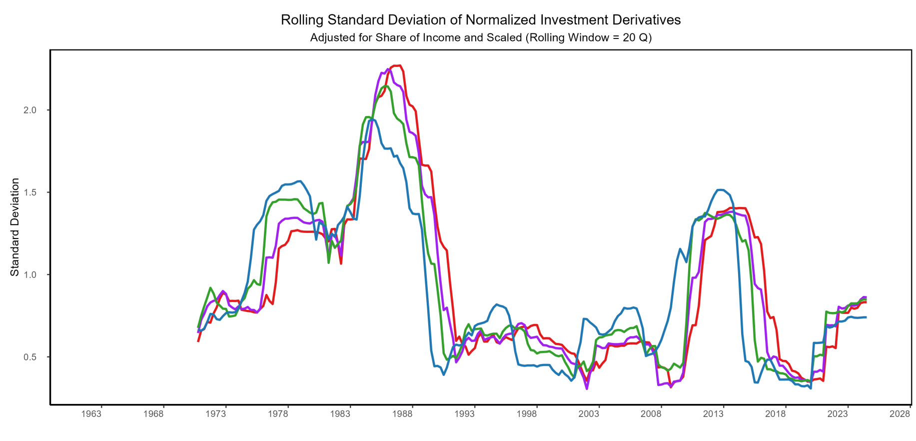

Figure 11: *Rolling Standard Deviations of Normalized Investment Derivatives*: This chart illustrates the increasing variability with each higher-order derivative. The observed pattern reflects that cyclical changes in investment are persistent, systematic, and non-decaying.

Table 2 summarizes the scaled correlations between derivative pairs for normalized consumption (C^*) and normalized investment (I^*). The high correlations between successive derivative pairs (e.g., $dC*/dt$ and d^2C*/dt^2) suggest bounded and persistent growth. As we have established, the higher-order correlations (i.e., d^3C*/dt^3 and d^4C*/dt^4) are slightly lower when correlated with the first derivative and second derivative, indicating that higher-order derivatives begin to operate in a somewhat independent manner, yet they remain highly correlated with each other. While this feature is indicative of exponential change and sinusoidal properties, it is also consistent with more complex systems where multiple frequencies are superimposed. Notably, the correlations diminish slightly as the derivative order increases (for the first and second derivatives), but the overall structure remains consistent with the theory of diminishing marginal returns.

Table 2: *Scaled Correlations Between Derivative Pairs of Normalized Consumption (C^*) and Normalized Investment (I^*)*

Derivative Pair	**Consumption (C^*)**	**Investment (I^*)**
(d/dt) and (d^2/dt^2)	0.724	0.726
(d/dt) and (d^3/dt^3)	0.375	0.393
(d/dt) and (d^4/dt^4)	0.179	0.393
(d^2/dt^2) and (d^3/dt^3)	0.849	0.849
(d^2/dt^2) and (d^4/dt^4)	0.594	0.594
(d^3/dt^3) and (d^4/dt^4)	0.903	0.903

Table 2 shows the high correlations between successive derivatives, which are consistent with bounded growth rates. The lower correlations between lower-order and higher-order derivatives, however, indicate that the derivatives are not merely tracking each other but exhibit a degree of independence, suggesting that the influence of earlier changes on later changes diminishes.

ARIMA modeling of the changes in income-normalized consumption (dC^*/dt) and income-normalized investment (dI^*/dt) provides further support for our findings. The ARIMA(3,0,0)(1,0,0) model for dC^*/dt yielded a high pseudo-R^2 of 0.708, reflecting robust autoregressive properties and stable, persistent oscillations. This supports our claim that the observed cycles are not random or the product of exogenous factors but are instead a stable part of the system. Similarly, the ARIMA(2,0,2)(1,0,2) model for dI^*/dt achieved a pseudo-R^2 of 0.763, confirming the cyclical stability of investment growth. These results align with the hypothesis that as resources shift between consumption and investment, the additional utility gained diminishes, creating a consistent, bounded trade-off between the two. Given the inverse relationship between consumption and investment, this sets both relative levels within predictable ranges, further showing that the system operates within stable parameters.

Additional robustness tests, including Vector Autoregression (VAR), spline regression, and a manual DSGE model, further confirmed the stability and persistence of these oscillations. Specifically:

First, the VAR Impulse Response Functions demonstrated significant and prolonged impacts of shocks, with dI^*/dt responding to dC^*/dt shocks for up to eight periods.[41]

Second, spline regression provided a close fit to the data, with an R^2 of 0.82 and a Residual Sum of Squares (RSS) of 14.5.[42]

Third, the DSGE model simulated diminishing marginal returns, showing convergence towards equilibrium after 12 periods.[43]

These findings challenge traditional business cycle theories, which emphasize exogenous shocks as the primary drivers of fluctuations. Our results suggest that these oscillations are not merely transient responses to external forces; instead, they represent a persistent, systematic, and non-random phenomenon inherent to the system's dynamics, contributing significantly to long-term economic growth. However, the fact that these higher-order derivatives do not correlate well with the original series suggests that while the immediate rates of change exhibit volatility, the fundamental structural relationships among rate, acceleration, and jerk persist over time. This stability aligns with the principles of diminishing marginal utility, where incremental changes become progressively less impactful even as fluctuations occur.

Next, we investigated third-degree polynomial regression to explore localized trends and evaluate potential parabolic, exponential, or sinusoidal characteristics. We segmented the data into 10-year intervals which we consider in alignment with medium-term economic cycles to capture temporal variations while ensuring robust model fits. We optimized a third-degree polynomial model ($f(x) = ax^3 + bx^2 + cx + d$), which revealed mixed adjusted R^2 values across segments, ranging from -0.04 to 0.45, with some segments exhibiting significant higher-order coefficients ($p < 0.001$). This mix of results, particularly the significant higher-order coefficients in some segments, supports the

[41] VAR models capture linear interdependencies among multiple time series. See [24].
[42] Spline regression captures non-linear trends through piecewise polynomials. For an overview, see [14].
[43] DSGE models represent economic principles through stochastic simulations. See [9].

hypothesis of a sinusoidal component in the oscillations and highlights how diminishing marginal utility forces a trade-off between consumption and investment.

In addition to analyzing consumption and investment derivatives, we examined the corresponding behavior of R^*_{3MTB} (*3MTB* minus the year-over-year change in GDP Proxy) and its derivatives to assess whether it behaves consistently as an income-normalized proxy for R, exhibiting similar patterns to those observed in dC^*/dt. The correlations among the first, second, and third derivatives of R_{3MTB} are substantially similar to those observed for dC^*/dt and its derivatives. Table 3 presents the scaled correlations between R^*_{3MTB} and its first, second, and third derivatives. The high correlations between the first and second derivatives, and second and third derivatives, of R^*_{3MTB} suggest consistency with the observed behavior of dC^*/dt and its derivatives, reinforcing the validity of the latter as a proxy for R^*.

Table 3: *Scaled Correlations Between R^*_{3MTB} and Its Derivatives*

Derivative Pair	**Scaled Correlation**
R^*_{3MTB} and dR^*_{3MTB}/dt	0.7281
R^*_{3MTB} and $d^3R^*_{\mathrm{3MTB}}/dt^3$	0.3962
$d^2R^*_{\mathrm{3MTB}}/dt^2$ and $d^3R^*_{\mathrm{3MTB}}/dt^3$	0.8563

Table 3 shows the high correlations between the first and second derivatives, and the second and third derivatives, of R^*_{3MTB}, demonstrating consistency with the observed behavior of dC^*/dt and its derivatives and reinforcing the validity of the latter as a proxy for R^*.

The strong correlations between the derivatives of R^*_{3MTB} and their ordinal correspondence with those found in analyzing the NIE Model Variables suggest that R^*_{3MTB} exhibits bounded and persistent oscillatory behavior similar to consumption and investment, reinforcing the conclusion that common forces may act as a bound on market interest rates, consumption, and investment. This similarity further supports the theory that the balance between consumption and investment follows a consistent adjustment process characteristic of an endogenous business cycle, as demonstrated in Section 4.5 (*Time Preference as the Income Allocation Mediator*). The close alignment between the ARIMA-based insights and the analysis of R^*_{3MTB} derivatives further strengthens this conclusion. This evidence strongly supports constructing our Augmented NIE Model Business Cycle using only the Augmented NIE Augmented NIE Model Variables to capture the interplay of these oscillatory forces.

7.5 R as Mediator

Having established the persistent oscillatory behavior of consumption and investment, we now present empirical evidence that directly supports the mediating role of R, the time preference for consumption, in governing the dynamic equilibrium between consumption and investment, which we demonstrated in Section 4.5 (*Time Preference as the Income Allocation Mediator*).

Figure 12 (*Correlation of Income and Investment Normalized for Consumption*) provides a scatter plot of the growth rates of income and investment over the Full Study Period (1962 Q1 - 2024 Q3), normalized for consumption (C). This chart empirically validates the Consumption-Normalized Identity of Income and Investment, confirming that changes in investment and income evolve in perfect tandem when expressed in terms of C.

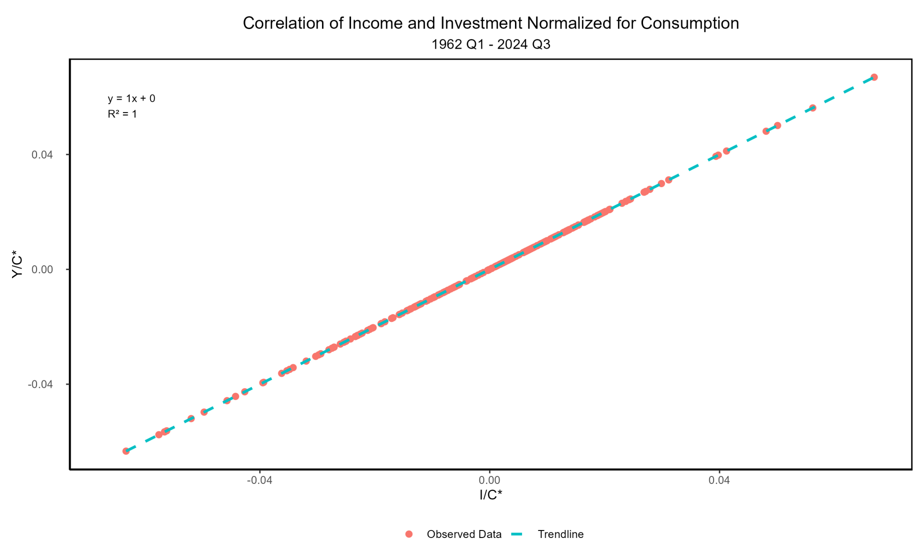

Figure 12: *Correlation of Income and Investment Normalized for Consumption:* This chart empirically confirms that income and investment evolve in tandem when normalized for consumption. The perfect linear fit ($y = 1x + 0$) and $R^2 = 1$ validate the Consumption-Normalized Identity of Income and Investment ($d(Y/C)/dt = d(I/C)/dt$), reinforcing the role of R in dynamically allocating income between consumption and investment to maintain stable disequilibrium.

This empirical result substantiates the key conclusion of Section 4.5: that R functions as the mechanism ensuring continuous and efficient reallocation of income between consumption and investment. The perfect one-to-one relationship between marginal changes in investment and income, when normalized for consumption, highlights how R governs this trade-off in alignment with the principles of efficient disequilibrium. Rather than being driven by external shocks, the business cycle exhibits an internally regulated structure, in which R dynamically governs the reallocation of income between consumption and investment. This regulation ensures that investment fluctuations remain proportionally aligned with income growth, preventing destabilizing divergences. The observed one-to-one relationship between $d(Y/C)/dt$ and $d(I/C)/dt$ confirms this internal mechanism, demonstrating that shifts in investment do not introduce instability but are instead absorbed through the systematic trade-off mediated by R.

7.6 Analysis of Normalized Interest Rates and the Normalized Consumption Derivative

To empirically validate the theoretical claim that time preference for consumption influences the cost of borrowing, we examined the relationship between income-normalized interest rates and the year-over-year change in income-normalized gross consumption (dGC^*/dt). This analysis is essential because the Augmented NIE Model Business Cycle incorporates R, the pure rate of interest (as well as PRI), and we aim to demonstrate that a robust proxy for R can be derived directly from its relationship to consumption behavior.

We analyzed three candidate interest rates: the 3-Month Treasury bill rate (3MTB), the commercial bank prime rate (Prime), and the U.S. 10-year Treasury bond rate (10YB). To isolate the influence of time preference, each rate was income-normalized by subtracting the year-over-year change in income (dY/dt), allowing for a direct comparison with dGC^*/dt. Across all three rates, we observed consistently high correlations: $R^2 = 0.792$ for dGC^*/dt with 3MTB* (Figure 13 (*Normalized 3-Month T-Bill Rate and Normalized Consumption Derivative*)); $R^2 = 0.744$ with Prime* (Figure 15 (*Normalized Prime Rate and Normalized Consumption Derivative*)); and $R^2 = 0.759$ with 10YB* (Figure 16 (*Normalized 10-Year Bond Rate and Normalized Consumption Derivative*)). These findings provide strong empirical confirmation of the theoretical linkage between time preference and the cost of borrowing.

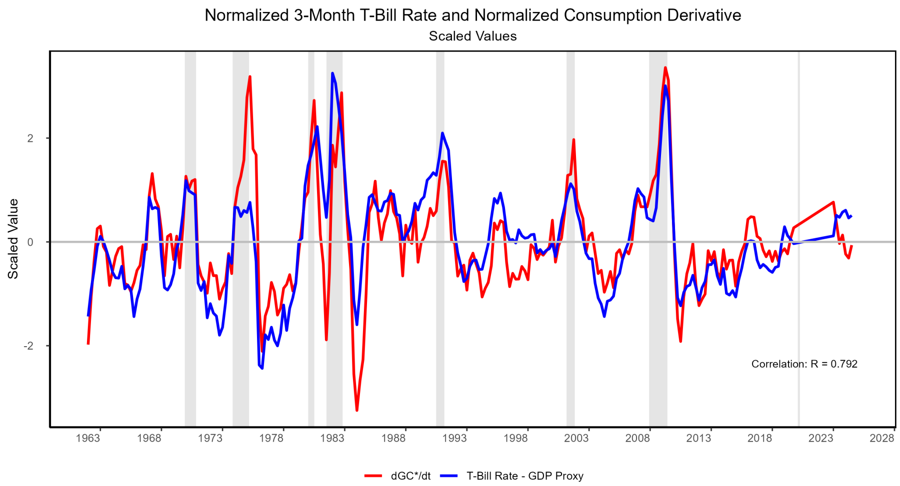

Figure 13: *Normalized 3-Month T-Bill Rate and Normalized Consumption Derivative:* This chart demonstrates the significant correlation between the rates of change of income-normalized 3MTB and income-normalized consumption (dGC^*/dt), supporting the theoretical connection between time preference and this risk-free rate.

To further validate the synchronicity between dGC^*/dt and the income-normalized 3MTB rate, we conducted a lagged correlation analysis. The results, shown in Figure 14 (*Lagged Correlation between Normalized 3-Month Treasury Bill Rate and Normalized Gross Consumption Derivative (dGC^*/dt)*), confirm an immediate and significant correlation, indicating that changes in consumption time preference are directly reflected in borrowing costs with no meaningful time lag.

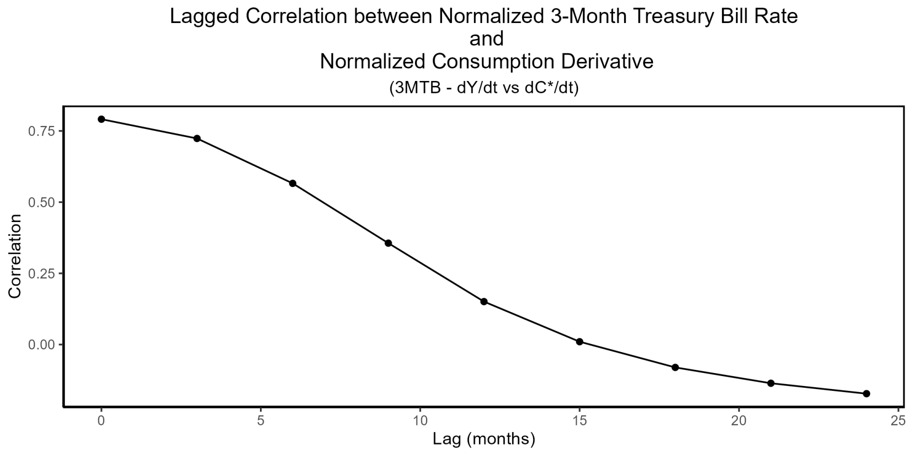

Figure 14: *Lagged Correlation between Normalized 3-Month Treasury Bill Rate (3MTB - GDP Proxy) and Normalized Gross Consumption Derivative (dGC^*/dt):* This chart confirms the immediate and significant correlation between the two income-normalized variables, further validating the linkage between time preference and this rate.

Figures 15 (*Normalized Prime Rate and Normalized Consumption Derivative*) and 16 (*Normalized 10-Year Bond Rate and Normalized Consumption Derivative*) exhibit similarly strong alignments between dGC^*/dt and income-normalized Prime and income-normalized 10YB, reinforcing the consistency of this relationship across different proxies for borrowing costs.

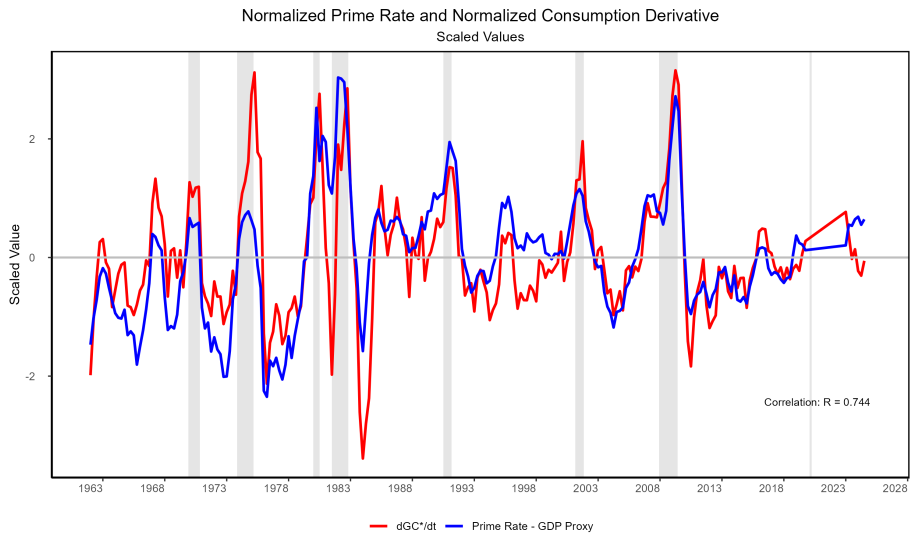

Figure 15: *Normalized Prime Rate and Normalized Consumption Derivative:* This chart highlights the strong correlation between Prime and dGC^*/dt, with both normalized for income changes, reinforcing the connection between changes in consumption and borrowing costs.

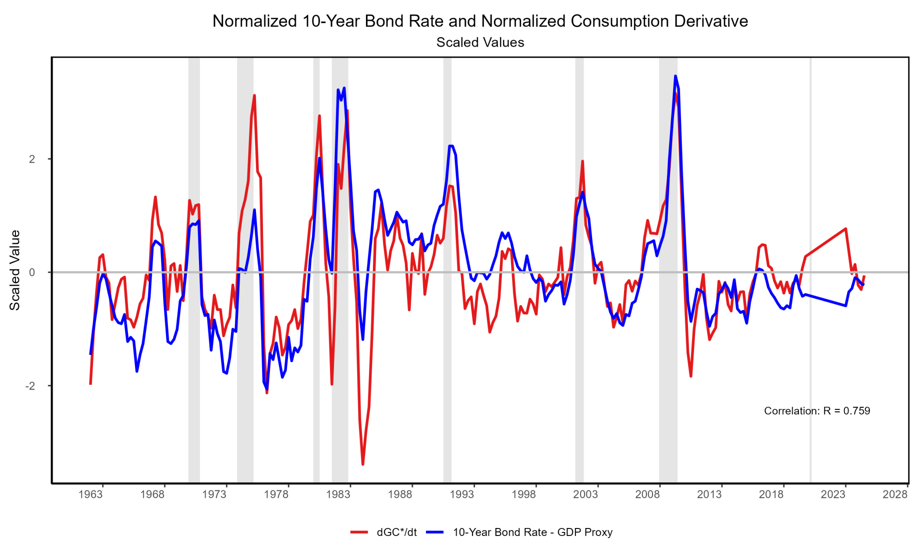

Figure 16: *Normalized 10-Year Bond Rate and Normalized Consumption Derivative:* This chart shows the strong correlation between 10YB and dGC^*/dt, consistent with the results for 3MTB and Prime rates.

The consistently strong correlations across all three candidate rates, along with the immediate synchronicity shown in the lagged correlation analysis, reinforce the fundamental role of time preference in shaping borrowing costs. This evidence lays the foundation for constructing an Augmented NIE Model Business Cycle that relies exclusively on the NIE Model Variables (Y, C, and I), without requiring exogenous interest rate inputs.

To further validate the link between time preference and borrowing costs, we analyzed the relationship between the income-normalized 3-Month Treasury bill rate (3MTB*) and the change in income-normalized consumption (dC*/dt). The small gap between these two variables, which we call the Consumption Rate Discrepancy (CRD), is shown in Figure 17 (*Consumption Rate Discrepancy and Y/Y Change in GDP Proxy*). Although we could have chosen any of the previously tested interest rates as a proxy for the pure interest rate (R), we selected the 3MTB* as our proxy for R* due to its nature as a short-term, nearly risk-free rate, and its reflection of substantial government borrowing. However, it's important to note that the 3MTB is sensitive to Federal Reserve policy, which can introduce distortions relative to the theoretical R construct, thus causing the CRD.

The cumulative CRD, calculated as the difference between the integrals of the income-normalized 3-Month Treasury bill rate (3MTB*) and the change in income-normalized consumption (dC^*/dt), is not shown in this paper. However, the cumulative CRD became increasingly negative from 1963 to 1980, a period marked by sustained and rising inflation. Subsequently, it gradually returned to zero over the next 24 years, which were characterized by lower, disinflationary price changes. On a quarterly basis, the CRD tends to strengthen during periods of economic weakness, when dC^*/dt overshoots 3MTB*, and weaken during periods of economic strength, when dC^*/dt undershoots 3MTB*. This suggests that the CRD behaves counter-cyclically, expanding during downturns and contracting during economic recoveries.

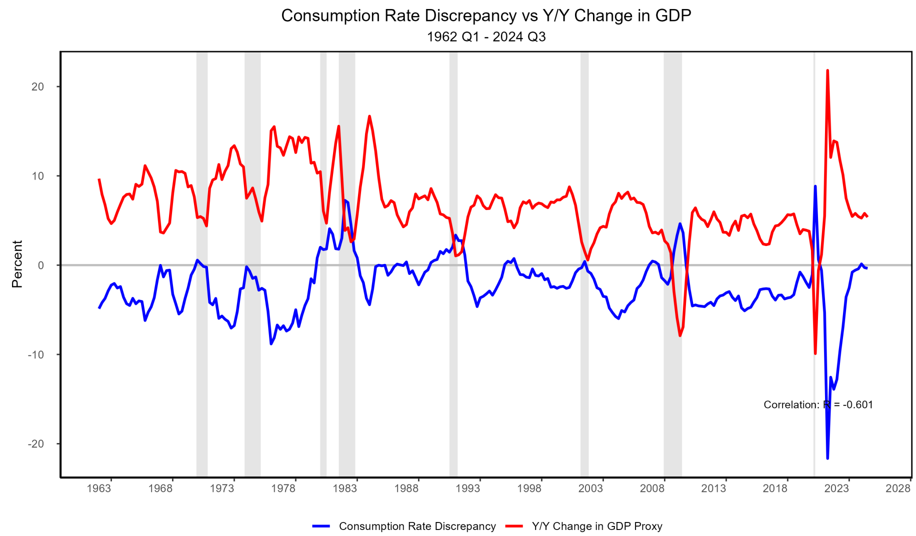

Figure 17: *Consumption Rate Discrepancy vs Y/Y Change in GDP Proxy:* This chart shows that the CRD strengthens as the income growth rate declines, with a notable negative correlation ($R = -0.595$) between the Consumption Rate Discrepancy (CRD) and changes in the GDP Proxy.

A plausible explanation for these dynamics is that during periods of slower economic growth, the time preference for consumption increases relative to income, effectively shifting the consumption curve rightward. This behavior reflects a reduced willingness of consumers to defer consumption in favor of investment. Conversely, during periods of rapid economic growth, the time preference for consumption decreases, shifting the curve leftward as consumers allocate more income toward future consumption through investment. These shifts are consistent with the theoretical role of R, the pure interest rate, in balancing consumption and investment through time preference.

To assess whether including income fluctuations improves the correlation between our R_{3MTB} proxy and consumption behavior, we incorporated the year-over-year change in our GDP Proxy into the analysis. Our objective was to determine how changes in income-normalized consumption and income (both endogenous variables measured by the GDP Proxy) influence borrowing costs, represented by the R_{3MTB} proxy. We employed an optimized rolling window approach, which means that we used a subset of data that advances through the full data set for each calculation, to account for potential shifts in the relationship over time. The model used a regression framework to estimate the coefficient (k value) that represents the influence of income changes on borrowing costs. The final model is expressed as: *Modeled* $R^* = dC^*/dt + k \cdot dY/dt$, where dC^*/dt represents the change in income-normalized consumption, dY/dt represents the change in income (GDP Proxy), and k is the estimated coefficient that optimally captures the influence of income changes on borrowing costs. After testing various rolling window sizes, we determined that the optimal period was 20 quarters, yielding a maximum average correlation of $R^2 = 0.917$ and an average k value of 0.893, indicating that changes in income-normalized consumption and income significantly determine borrowing costs over time, supporting the idea that the interest rate is endogenous to the model.

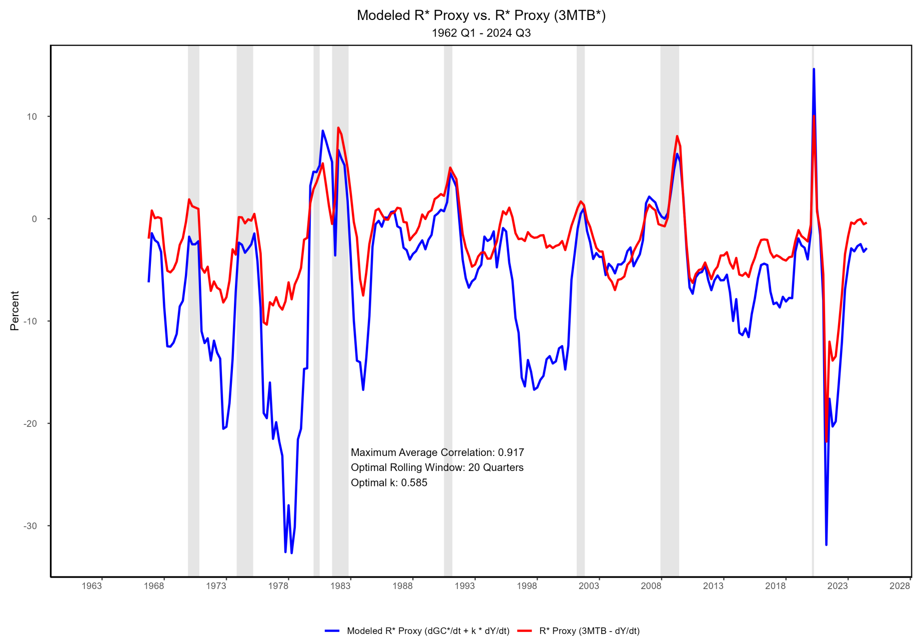

Figure 18: *Modeled* R^* *Proxy vs. Normalized 3-Month T-Bill Rate:* This chart compares the modeled R^* proxy with the normalized 3MTB*, demonstrating a high correlation ($R^2 = 0.917$) and validating the predictive capability of the model.

The strong correlation supports the form of the modeled $R_{3\text{MTB}}$ proxy, which is a key element of the Augmented NIE Model Business Cycle. This result provides compelling empirical support for the Augmented NIE Model Business Cycle by demonstrating that borrowing costs can be explained solely by endogenous consumption and income dynamics, without the need for exogenous interest rate inputs. This endogenous derivation of R from within the model's own dynamics reinforces the model's predictive capability and its ability to capture the fundamental drivers of business cycle fluctuations.

7.7 Building the Augmented NIE Model Business Cycle

Although the cyclical nature of economic progress is evident in real-world data, traditional analyses often obscure the underlying dynamic interactions among key components that drive it. Conventional economic approaches struggle to disentangle the interdependent movements of consumption (C), investment (I), and income (Y). By normalizing these variables, we can isolate their intrinsic dynamics to reveal the structural forces that govern economic oscillations. The Augmented NIE Model Business Cycle model thus establishes a direct relationship between the Augmented NIE Augmented NIE Model Variables and their income-normalized counterparts, demonstrating how their dynamic interactions shape the business cycle in a systematic manner.

The Augmented NIE Model Business Cycle elucidates how the economy adeptly meets the core tenets of our economic philosophy: that economic activity exists to facilitate consumption, and for this to occur sustainably, income (Y), consumption (C), and investment (I) must grow in harmony across business cycles. The model weaves together both raw and normalized Augmented NIE Augmented NIE Model Variables into a coherent, theoretically sound framework with immediate practical applications. Within this structure, we can clearly identify and verify the efficiency benchmarks proposed by our theory, using real-world data to showcase their validity. This methodology not only effectively connects theoretical economics with practical, data-informed insights into economic dynamics but also lays a solid groundwork for policy-makers.

7.7.1 Theoretical Foundations for the Augmented NIE Model Business Cycle

We will build our Augmented NIE Model Business Cycle on three foundational underpinnings, namely, time preference (R), marginal utility, and the Central Growth Rate (CGR).

Time Preference. In our theory, the time preference for consumption, represented by the pure interest rate (R), mediates the allocation of income between consumption and investment by determining the extent of consumption deferral. In an ideal economy with a consistent mean secular potential return on investment (PRI) and mean secular pure interest rate (R), R dictates the degree of consumption postponement and income investment as income (dY/dt) and investment ($dI\#/dt$) oscillate in response to internal cyclical dynamics. However, in practice, R and PRI are influenced by a range of external factors, including monetary policy interventions by central banks, fiscal policy, and broader economic shocks such as financial crises and technological disruptions. These external influences introduce variability into the oscillatory patterns of R and PRI, resulting in deviations from their theoretical trajectories. As a result, real-world R and PRI dynamically adjust to evolving economic conditions, both internal and external, generating corresponding fluctuations in cyclical consumption deferral and income investment that shape activity within and across business cycles.

A significant challenge in modeling the business cycle lies in the absence of a definitive market interest rate that represents R, the time preference for consumption and the cost of borrowing.[44] As already discussed, economists often rely on commercial bank and U.S. treasury interest rates to represent R. The three-month U.S. Treasury Bill (3MTB), reflecting a short-term, risk-free borrowing cost for the U.S. Government is a common choice. While the analysis in Section 7.6 demonstrates a strong correlation ($R^2 = 0.792$) between dGC^*/dt and 3MTB*, variations in 3MTB, driven by policy interventions or market conditions and demand from liquidity or safety concerns not directly associated with the time preference for consumption may introduce deviations not intrinsic to the pure interest rate.

There is no theoretical basis requiring the cost of borrowing (R) to depend on external interest rates that cannot be fully expressed within the confines of the Augmented NIE Model Variables themselves, and, based on the analysis described earlier in this Section 7, the rate of change of consumption $dC^{\#}/dt$ commends itself as the prime candidate for R.

A significant theoretical result emerges when we model the income-normalized pure interest rate (R^*) as dC^*/dt, the rate of change of income-normalized consumption. Substituting dC^*/dt for R^* in Equation (12) ($R^* = PRI^* - dI^*/dt$) yields the elegant implication that all potential returns on

[44] See [20] for a discussion of the difficulties in measuring the natural rate of interest.

investment (PRI^*) are fully realized in the rate of income growth (dY/dt):

Step 1: Substitute dC^*/dt for R in Equation (12):

$$\frac{dC^*}{dt} = PRI^* - \frac{dI^*}{dt}$$

Step 2: Using Equation (10) ($dC^*/dt = -dI^*/dt$), substitute dC^*/dt for $-dI^*/dt$:

$$\frac{dC^*}{dt} = PRI^* + \frac{dC^*}{dt}$$

Step 3: Simplify:

$$0 = PRI^*$$

The equation $PRI^* = 0$ reflects the income-normalized relationship between potential returns and growth dynamics. Since income-normalizing the Augmented NIE Augmented NIE Model Variables removes the absolute level of income (Y) from the analysis, the equality $PRI^* = 0$ implies that the actual potential return on investment (PRI) aligns with the economy's realized income growth rate (dY/dt). This follows because normalization isolates cyclical deviations from secular trends, ensuring PRI dynamically tracks dY/dt rather than diverging as a standalone variable.

While this derivation ensures theoretical consistency for the assumption $dC^*/dt = R^*$ within the model, it does not conclusively prove the equality. Instead, it demonstrates that the relationship is logically coherent when viewed through the lens of income normalization and cyclical equilibrium dynamics. Given this coherence and the empirical alignment observed in prior sections, dC^*/dt emerges as the most consistent and theoretically justified proxy for R^* in our model.

Marginal Utility. Diminishing marginal utility, a well-established principle in economic theory, is a cornerstone of the Augmented NIE Model Business Cycle. As consumption increases, the additional satisfaction or utility derived from consuming each additional unit of income decreases, ensuring that the rate of growth of consumption as a proportion of income is bounded unless the rate of income growth is continuously accelerating, a proposition that is inconsistent with our assumption of a continually progressing economy (CPE) and real-world experience. The concept of diminishing marginal utility of income consumption implies that each increment of ΔY allocated to C that increases its share of income (C/Y) causes C/Y to increase in declining increments as it approaches its upper bound of 1.0.

Mathematically, the rate of change of income-normalized consumption (dC^*/dt) is given by:

$$\frac{dC^*}{dt} = \frac{dY/dt \cdot (1 - C^*)}{Y} \tag{18}$$

Equation (18)[45] shows that as the consumption share of income (C^*) increases, the rate of change of C^* diminishes, constrained by the term $(1-C^*)$. As C^* approaches 1.0, the rate of growth slows, reflecting the diminishing marginal utility.

The relationship between C/Y and ΔY reinforces the principle of diminishing marginal utility confirmed by our statistical analysis. Starting from any initial ratio of consumption to income (C/Y), each additional unit of ΔY allocated to consumption increases C/Y by progressively smaller amounts, constrained by its upper limit of 1.0.[46] This diminishing increase reflects the declining utility gained from additional consumption. As C/Y approaches its cap, this dynamic ensures that oscillations in consumption and investment are bounded, governed by diminishing marginal returns. A similar logic applies to investment, as the marginal efficiency of capital declines with increasing investment, suggesting that both cycles exhibit stable and predictable patterns. This behavior is both a mathematical outcome and an economic reflection of resource trade-offs.

The Central Growth Rate. As will be demonstrated in Section 7.7.3 (*The Augmented NIE Model Variable Relationships*), the Central Growth Rate (CGR) acts as a cap on the rate of change of consumption relative to income, providing a natural bound to the oscillatory dynamics of the business cycle. These bounded, non-parabolic oscillations reflect the interplay between income growth and consumption, creating a stable system in which investment increases to meet future

[45] For the derivation of this equation, see Appendix B (*Mathematical Derivation of the Rate of Change of Income-Normalized Consumption.*

[46] We note that in our Full Study Period, GC^* exceeded 1.0 for two consecutive quarters during the Great Recession, indicating that consumption during that brief period extended beyond income to the existing capital base. Since our focus is on the dynamics of growing economies, this aberration does not affect our analysis and should be seen as supporting the proposition that excessive consumption is anathema to growth.

needs once current needs have been satisfied.[47]

Implications for the Augmented NIE Model Business Cycle. The theoretical foundations outlined above—time preference, diminishing marginal utility, and the CGR as a cap—provide the critical components for constructing the Augmented NIE Model Business Cycle. These relationships establish a bounded and stable framework in which all income is considered either consumed or invested, ensuring that the economy operates within the constraints of time preference and marginal utility. In the next section, we will combine these components to construct the full Augmented NIE Model Business Cycle.

7.7.2 The Augmented NIE Model Business Cycle

With the assumption that $dC^*/dt = R^*$ and utilizing the CGR as described, we can now use the relationships derived from the Augmented NIE Model Variable equations defined in Section 4 to build the complete Augmented NIE Model Business Cycle shown in Figure 19. This step uses the core mathematical framework, which has been derived from our assumptions, to create a model against which we can interpret real-world data.

[47] See Section 4.5 (*Time Preference as the Income Allocation Mediator*).

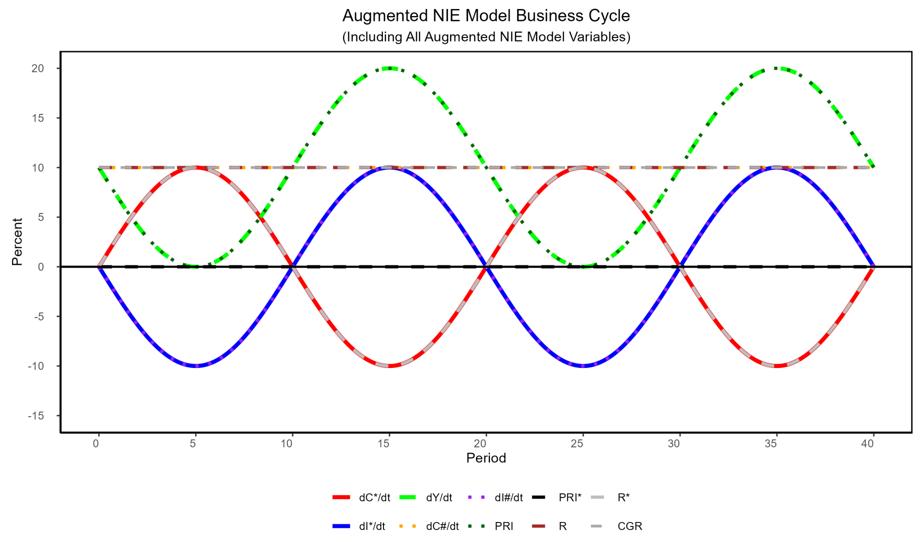

Figure 19: *Augmented NIE Model Business Cycle:* This chart shows the interaction of all income-normalized and income-adjusted Augmented NIE Augmented NIE Model Variables. Note that dC^*/dt and R^* are aligned, PRI^* is aligned at the Zero Line, PRI is aligned with dY/dt, dI^*/dt and $dI^{\#}/dt$ are aligned, and $dC^{\#}/dt$ and R are aligned at the CGR.

In the Augmented NIE Model Business Cycle, income is allocated between consumption and investment based on the dynamic interplay between the time preference for consumption (R) and the potential return on investment (PRI). To analyze this interplay without the direct influence of income growth, we normalize consumption and investment by expressing them as proportions of income, setting dY/dt as the Zero Line on the Y-axis (Percent). This Zero Line serves as a baseline for cyclical interactions among income-normalized consumption, investment, PRI^*, and R^*. However, it does not represent absolute levels or imply near-zero values for these rates of change, as dY/dt itself fluctuates over time. The alignment of modeled PRI^* at the Zero Line reinforces the theoretical role of R in balancing consumption and investment to optimize present and future consumption.

Figure 19 provides a critical visualization of these dynamics, illustrating how the income-normalized variables interact within the context of income growth and allocation. It includes dY/dt, dC^*/dt, dI^*/dt, R^*, PRI^*, $dC^{\#}/dt$, and $dI^{\#}/dt$, as well as R and PRI, demonstrating their relationships to each other, the Central Growth Rate (CGR), and the Zero Line. Notably,

PRI aligns with dY/dt, R^* with dC^*/dt, $dC^{\#}/dt$ with the CGR line, dI^*/dt with $dI^{\#}/dt$, and PRI^* with the Zero Line.

The oscillatory dynamics shown in Figure 19 exhibit several key properties.

First, the variables dC^*/dt, R^*, dI^*/dt, and $dI^{\#}/dt$ oscillate around the Zero Line, with the first two moving in direct opposition to the latter two, maintaining the Zero-Sum Dynamic ($dC^*/dt = -dI^*/dt$).

Second, the peaks and troughs of dC^*/dt coincide with the troughs and peaks of dY/dt and PRI. When dC^*/dt reaches its peak, both dY/dt and PRI equal zero.

Third, the peaks and troughs of dI^*/dt and $dI^{\#}/dt$ align with the peaks and troughs of dY/dt and PRI, reinforcing their cyclical interaction.

Fourth, the amplitudes of dC^*/dt, dI^*/dt, R^*, PRI, and dY/dt are strictly equal, with their equality maintained through the income-share adjustment that scales the income-normalized rates of change of consumption and investment.

Fifth, the crossings of dC^*/dt and R^* above and below the Zero Line correspond with the crossings of dY/dt and PRI below and above the CGR line.

Finally, the crossings of dI^*/dt and $dI^{\#}/dt$ above and below the Zero Line correspond with the crossings of dY/dt and PRI above and below the CGR line.

Figure 19 also shows the following bounds on the Augmented NIE Augmented NIE Model Variables.

First, the Central Growth Rate (CGR) serves as an upper limit on dC^*/dt, R^*, dI^*/dt, and $dI^{\#}/dt$, while the opposite of the CGR (not shown) functions as a lower bound on these variables.

Second, the Zero Line represents the lower limit for dY/dt and PRI, which is their minimum value in a CPE with a constant CGR.

Third, the upper limit of dY/dt and PRI is precisely twice the CGR, defining the maximum observed amplitude for these variables.

7.7.3 The Augmented NIE Model Variable Relationships

Our observations on the roles of dY/dt, dGC^*/dt, dNI^*/dt, and the income shares of consumption (C/Y) and investment (I/Y) in Figure 19 can be clarified through formulaic expressions, emphasizing the centrality of the CGR and elucidating their dynamics.

First, the Zero-Sum Dynamic ensures that any increase in the share of income directed toward consumption comes at the expense of investment, and vice versa. This relationship is expressed as:

$$dGC^*/dt = -dNI^*/dt.$$

This fundamental principle underpins the cyclical reallocation of income within the system.

Second, the peak of income-normalized consumption growth occurs relative to the CGR, scaled by the trough of dY/dt and the share of income allocated to consumption. Since the peak occurs when $dY/dt = 0$, this relationship can be expressed as:

$$dGC^*/dt_{\text{peak}} \approx (C/Y) \cdot (\text{CGR} - dY/dt_{\text{trough}}).$$

Rewriting, we obtain:

$$dGC^*/dt_{\text{peak}} \approx dC^{\#}/dt \approx \text{CGR}.$$

Third, the peak of income-normalized investment growth occurs as the difference between the peak of dY/dt and $dC^{\#}/dt$, which itself peaks at the CGR. This is formalized as:

$$dI^*/dt_{\text{peak}} = dY/dt_{\text{peak}} - dC^{\#}/dt.$$

At its peak, investment growth satisfies:

$$dI^*/dt = dI^{\#}/dt \approx \text{CGR}.$$

Fourth, the peak of income growth reflects twice the CGR, consistent with the model's dynamics as discussed in Section 4.3.2:

$$dY/dt_{\text{peak}} = 2 \cdot \text{CGR}.$$

Finally, the income-normalized change in investment is equal to the income-share adjusted change in investment, with both oscillating around the Zero Line and peaking at the CGR. This is represented as:

$$dI^*/dt = dI^{\#}/dt.$$

We will demonstrate this fundamentally important relationship in Section 7.7.5 as Equation (19), relying in part on the CGR Proposition.

The Central Growth Rate (CGR) is a key reference point in the Augmented National Income Equilibrium Model Business Cycle, serving as a benchmark and a constraint that moderates the amplitudes of income-normalized consumption growth (dGC^*/dt) and income-normalized investment growth (dNI^*/dt) to ensure sustainable economic growth. Oscillations in income growth (dY/dt) influence the cyclical interactions between consumption and investment. The Zero-Sum Dynamic ($dGC^*/dt = -dNI^*/dt$) is essential for maintaining balance, ensuring that an increase in the share of income going towards consumption is offset by a corresponding decrease in investment, and vice versa. The model emphasizes dual efficiency, in which consumption growth is aligned with the CGR, and investment growth is dynamically modulated to align with the economy's potential output. The relationships within the Augmented NIE Model Business Cycle are endogenous and not the result of external shocks, providing a robust framework for analyzing both idealized and observed economic oscillations. In summary, key aspects of the model include the fact that the peak of income growth is equal to twice the CGR, and that the income-normalized and income-share adjusted changes in investment are equivalent, oscillating around the Zero Line and peaking at the CGR.

So far, we have treated the peaks in dC^*/dt as empirically observable phenomena. We now turn to the formal derivation of the peaks and troughs of dC^*/dt and dI^*/dt using the equations of the

Augmented NIE Model. From Equation (14), we know that:

$$CGR = \frac{dY}{dt} + \frac{dC^*}{dt} \approx \frac{dC^{\#}}{dt}$$

This relationship allows us to calculate the peaks and troughs of dC^*/dt and dI^*/dt within our framework.

The Peak of dC^*/dt: Start with the Central Growth Rate (CGR) Proposition.

Step 1: Rearrange the equation to express dC^*/dt in terms of the CGR and dY/dt:

$$\frac{dC^*}{dt} = CGR - \frac{dY}{dt}$$

Step 2: Define the peak of dC^*/dt as its maximum value within a business cycle, which occurs when the term being subtracted from the CGR is at its minimum.

Step 3: Note that, in the model of a Continually Progressing Economy (CPE), the minimum value of dY/dt within its oscillation is zero, which is the point at which income growth transitions from positive to negative (or *vice versa*).

Step 4: Substitute the minimum value of dY/dt, which is 0, into the rearranged CGR Proposition equation:

$$\frac{dC^*}{dt} peak = CGR - 0$$

Step 5: Simplify the equation:

$$\frac{dC^*}{dt} peak = CGR$$

This is a logical consequence of the CGR Proposition, and the definition of the minimum value of dY/dt in a *CPE*. Note that, at the peak of dC^*/dt, the Zero-Sum Dynamic ($dC^*/dt = -dI^*/dt$) ensures that dI^*/dt is at its trough.

The Trough of dC^*/dt: To calculate the trough of dC^*/dt, we start with its relationship to the income-share adjusted variable $dC^{\#}/dt$ and income growth dY/dt in the CGR Proposition.

Step 1: Rearrange Equation (14) as follows:

$$dC^*/dt = dC^\#/dt - dY/dt.$$

Step 2: The trough of dC^*/dt occurs when dY/dt is maximized because as can be seen from a rearrangement of the CGR Proposition ($dC^*/dt = CGR - dY/dt$) a higher dY/dt reduces dC^*/dt. From the proof in Section 4.3.2 (*Demonstrating that* CGR *is the Mean of* dY/dt *as it Oscillates*), we know that the maximum value of dY/dt is $2 \cdot dC^\#/dt$. Substituting this into the equation gives:

$$dC^*/dt_{trough} = dC^\#/dt - (2 \cdot dC^\#/dt).$$

Step 3: We can simplify this equation as follows:

$$dC^*/dt_{trough} = -dC^\#/dt.$$

Thus, at its trough, $dC^*/dt = -dC^\#/dt$. This result reflects the Zero-Sum Dynamic ($dC^*/dt = -dI^*/dt$), ensuring that any trough in consumption growth corresponds to a peak in investment growth.

The oscillatory symmetry of the model ensures that dC^*/dt cannot fall below $-dC^\#/dt$. This relationship holds under all conditions of a *CPE*.

First, when $dY/dt = 0$, we observe that:

$$dC^*/dt = dC^\#/dt,$$

which is always greater than $-dC^\#/dt$.

Second, at $dY/dt = \text{CGR}$ (the mean of income growth), the relationship is:

$$dC^*/dt = dC^\#/dt - \text{CGR} = 0,$$

which remains greater than $-dC^{\#}/dt$.

Finally, at all other points in the cycle, the constraint $|dY/dt| < 2 \cdot dC^{\#}/dt$ ensures that:

$$|dC^{*}/dt| < |2 \cdot dC^{\#}/dt|.$$

This condition preserves the structural limits of the oscillatory dynamics.

This theoretical result aligns closely with Figure 19, which illustrates these dynamics in a continually progressing economy (CPE). Furthermore, empirical validation from Figure 25 (*Y/Y Changes in GDP and Normalized Gross Consumption*) demonstrates that real-world data supports this conclusion by showing that $dC^{*}dt$ is bounded within the Y/Y Changes in GDP trend lines, which represent the CGR, reinforcing that consumption growth oscillates predictably within its bounds.

The Peaks and Troughs of dI^{*}/dt. The peaks and troughs of dI^{*}/dt follow directly from the Zero-Sum Dynamic expressed in Equation 10 ($dC^{*}/dt = -dI^{*}/dt$). This relationship implies that the behavior of dI^{*}/dt mirrors that of dC^{*}/dt but with opposite signs, ensuring that any change in consumption is met by a corresponding shift in investment.

The peak of dI^{*}/dt occurs when dC^{*}/dt is at its trough. Since the trough of $dC^{*}/dt = -dC^{\#}/dt$ occurs when dY/dt peaks at $2 \cdot dC^{\#}/dt$, it follows that the peak of $dI^{*}/dt = dC^{\#}/dt$ also occurs when dY/dt peaks at $2 \cdot dC^{\#}/dt$.

Similarly, the trough of dI^{*}/dt occurs when dC^{*}/dt is at its peak. Since the peak of $dC^{*}/dt = dC^{\#}/dt$ occurs when $dY/dt = 0$, the trough of $dI^{*}/dt = -dC^{\#}/dt$ also occurs when $dY/dt = 0$.

These results underscore the alignment of the oscillatory behavior of dC^{*}/dt and dI^{*}/dt within the bounds of Central Growth Rate (CGR) and its opposite ($-CGR$). At all times, the sum of these two income-normalized rates equals zero, ensuring the cyclical balance between consumption and investment.

These oscillations illustrate the essential roles of the Zero-Sum Dynamic ($dC^{/}dt = -dI^{/}dt$) and the Central Growth Rate (CGR) in providing a stable yet adaptive structure for the business cycle. The Zero-Sum Dynamic maintains the balance of income allocation between consumption and investment, while the CGR serves as the central value around which dY/dt and PRI oscillate and the upper and lower bounds constraining the fluctuations of $dC'dt$ and $dI'dt$. The formal structure of these relationships, derived from income normalization, is not merely theoretical but can be observed in real economic cycles, as we will now demonstrate.

7.7.4 Real-World Application of The Augmented NIE Model Business Cycle Dynamics

As we turn to the real-world application of the Augmented NIE Model Business Cycle, it is important to highlight the complementary insights offered by the income-normalized and unnormalized models. Because normalization removes the effects of absolute income levels over time, the income-normalized model reveals the persistent presence of the Zero-Sum Dynamic within and across business cycles. In contrast, applying the full model to real-world data illustrates how all the Augmented NIE Augmented NIE Model Variables interact within the broader economy, reflecting both the overarching trend of secular economic growth and the disruptions caused by recessions and economic downturns.

We can first apply the normalized Augmented NIE Model Business Cycle relationships to real world normalized data, taking dGC^*/dt and dNI^*/dt as they occurred. Figure 20 (Normalized Augmented NIE Model Business Cycle) illustrates the interactions among normalized consumption growth (dGC^*/dt), and normalized investment growth (dNI^*/dt), while aligning normalized potential return on investment (PRI^*) with the Zero Line and the normalized pure interest rate (R^*) with dGC^*/dt.

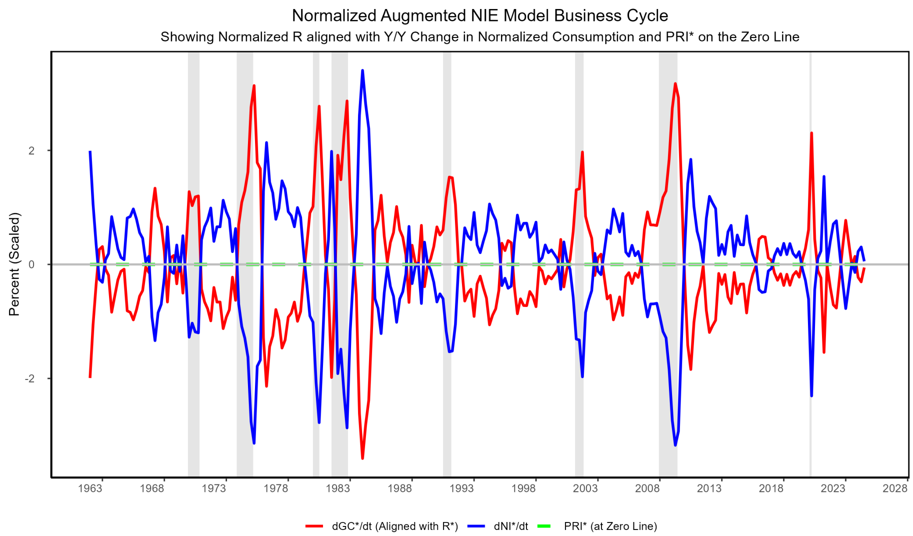

Figure 20: *Normalized Augmented NIE Model Business Cycle:* This chart illustrates the normalized Augmented NIE Model Business Cycle, showing the interactions among dGC^*/dt (aligned with R^*, dNI^*/dt, and PRI^*). The green line, PRI^*, represents the potential return on investment. It appears directly on the Zero Line, illustrating that in an ideal economy PRI^* is always equal to zero when normalized, indicating the total realization of potential return on investment within the constraint of time preference across cycles.

Next, we can extend the application of the Augmented NIE Model Business Cycle to real-world data from the Full Study Period, including actual values for income growth (dY/dt), consumption growth ($dC^{\#}/dt$), and investment growth ($dI^{\#}/dt$). Figure 21 (Augmented NIE Model Business Cycle Based on Real-World Data) presents this unnormalized perspective, enabling direct observation of critical economic phases such as recessions and expansions.

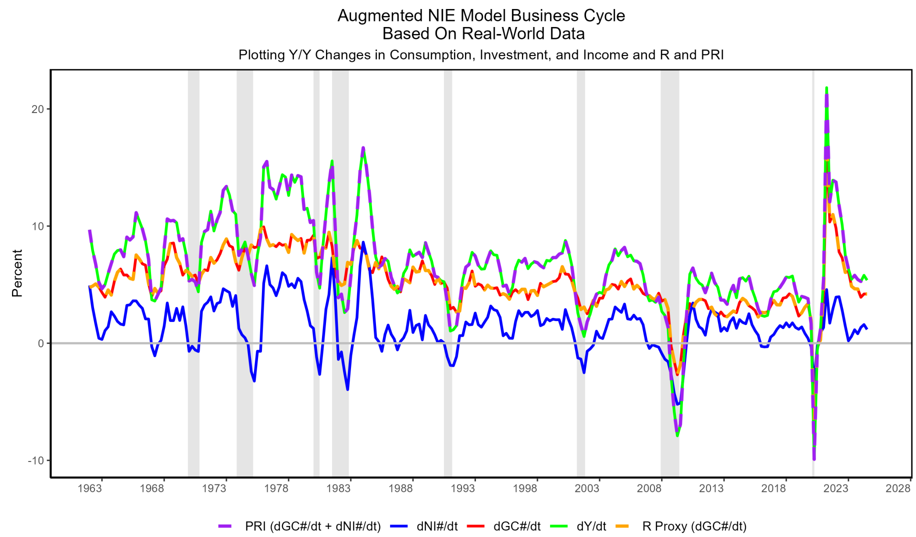

Figure 21: *Augmented NIE Model Business Cycle Based On Real-World Data:* This chart presents the full, unnormalized model of the Augmented NIE Model Business Cycle shown in Figure 20, including the income growth rate dY/dt, the consumption growth rate $dC\#/dt$, the investment growth rate $dI\#/dt$, the potential return on investment (PRI), and the pure interest rate (R).

Figures 20 and 21 offer complementary perspectives on the Augmented NIE Model Business Cycle. The income-normalized model highlights center-seeking oscillations of dGC^*/dt and dNI^*/dt around the Zero Line, illustrating the persistent presence of the Zero-Sum Dynamic in real-world data. In this framework, PRI^* remains aligned with the Zero Line, consistent with the theoretical expectation that the realized potential return on investment is fully accounted for within the income-normalized economy. In contrast, the unnormalized model reveals variations in absolute income and its allocation, pinpointing critical moments such as recessions, when $dI^{\#}/dt$ falls to zero and $dC^{\#}/dt$ equals $dY^{\#}/dt$. Together, these perspectives emphasize the dynamic interplay of consumption, investment, and income growth across the business cycle.

7.7.5 Structural Elements of the Augmented NIE Model Business Cycle

To validate the Augmented NIE Model Business Cycle, we will demonstrate, through mathematical analysis and reference to nominal and real data series, three fundamental structural elements.

First, the CGR line serves as a Benchmark of Optimal Consumption. The formulaic expression of the CGR in Equation (14), given by

$$CGR = dC^*/dt + dY/dt = dC^\#/dt,$$

represents the summation of income-normalized gross consumption and income growth. This relationship captures the fundamental balance between consumption and income within the system.

Second, the CGR line functions as an Oscillatory Boundary. The CGR acts as a cap on the rates of change of income-normalized consumption (dC^*/dt) and investment (dI^*/dt), reflecting the principle of marginal utility and enforcing cyclical stability.

Finally, the Zero Line serves as the Benchmark of Optimal Investment. The formulaic expression of the dynamic change in investment,

$$dNI^*/dt \approx dNI^\#/dt,$$

reveals that both the income-normalized and unnormalized investment derivatives oscillate around the Zero Line. This behavior illustrates the balance between investment activity and economic potential, as the system strives to optimize investment within the constraints of time preference and income growth.

The Central Growth Rate (CGR) as a Benchmark. Equation (14) ($CGR = dC^*/dt + dY/dt = dC^\#/dt$) expresses the fundamental balance between income-normalized consumption growth and income growth within the system. Since $dC^\#/dt$ is defined as their sum, it directly tracks the CGR, ensuring that consumption growth remains proportional to long-term income trends. Empirical evidence supports this formulaic relationship, as demonstrated in the following figures. Figure 22 shows the virtually perfect alignment of the trendlines for dY/dt and $dC^*/dt + dY/dt$ across the Full Study Period.

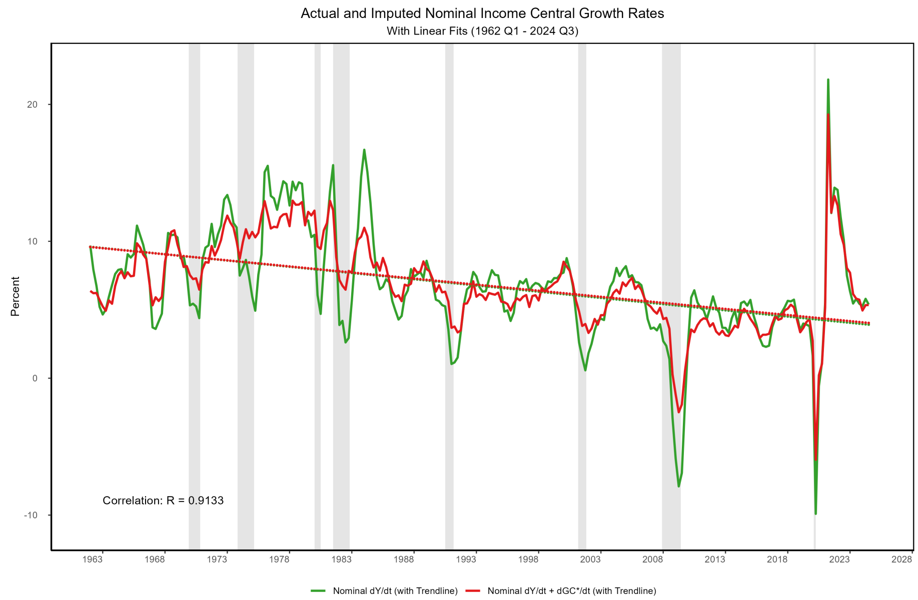

Figure 22: *Actual and Imputed Nominal Income Central Growth Rates:* This chart shows the near-perfect alignment of the actual and imputed Central Growth Rates (CGR) for the Full Study Period.

Similarly, Figure 23 shows the virtually perfect alignment of the trendlines for dY/dt, $dC^*/dt + dY/dt$, and $dC^{\#}/dt$ across the Full Study Period.

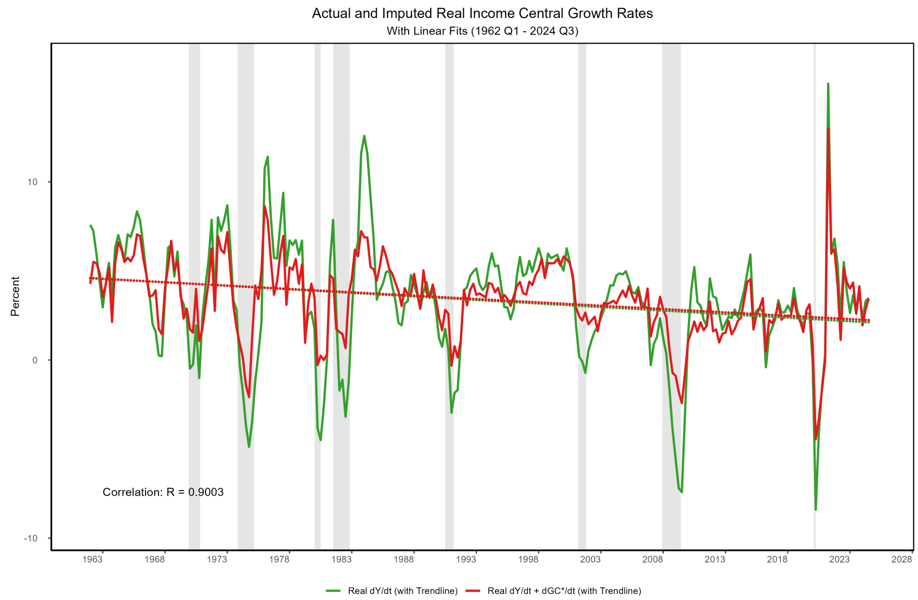

Figure 23: *Actual and Imputed Real Income Central Growth Rates:* This chart shows the near-perfect alignment of the actual and imputed real Central Growth Rates (CGR) for the Full Study Period.

We find a third confirmation of the validity of the CGR Proposition formula in Figure 24 (*Comparison of Consumption with the Central Growth Rate*), showing how closely (R=0.9961) the year-over-year change in income-adjusted consumption tracks $dC^*/dt + dY/dt$, which represents the income-share adjusted form of the income-normalized consumption derivative.

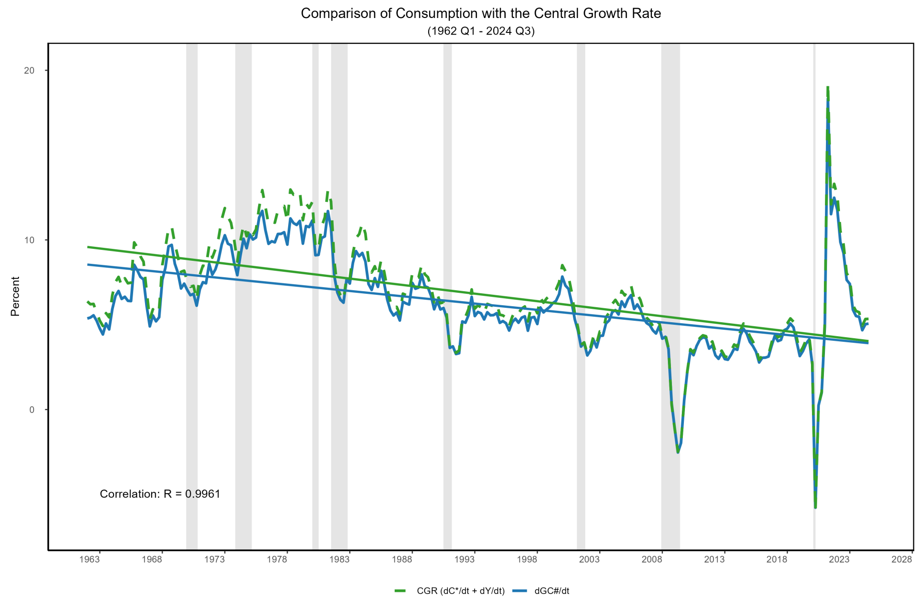

Figure 24: *Comparison of Consumption with the Central Growth Rate:* This chart shows the near-perfect alignment of the income-adjusted Gross Consumption growth rate (dGC/dt) with the imputed Central Growth Rates (CGR) for the Full Study Period, supporting the CGR Proposition.

While the theoretical model depicts a constant CGR for analytical clarity, the real-world data reveals considerable variability in the actual CGR. Nevertheless, the close alignment of the trend-lines demonstrates how $dC^{\#}/dt$ efficiently adjusts to track these real-world fluctuations, validating the model's core premise that consumption growth dynamically adapts to maintain optimal resource allocation even as economic conditions vary. This adaptive efficiency underscores the robustness of the Augmented NIE Model in explaining both idealized and actual business cycle dynamics.

The Central Growth Rate (CGR) as an Oscillatory Boundary. In our Augmented NIE Model Business Cycle, the CGR also acts as a boundary that caps the peaks and troughs of dC^*/dt and dI^*/dt. Figure 25 (*Y/Y Changes in GDP and Normalized Gross Consumption*) demonstrates how peaks in dGC^*/dt align with troughs in nominal dY/dt but do not usually reach the CGR (represented by the trendline), as called for by the model. This discrepancy arises because inflation introduces a persistent upward bias in nominal income growth, generally preventing it from falling to or below zero, which is a mathematical condition required for dC^*/dt to reach the CGR.

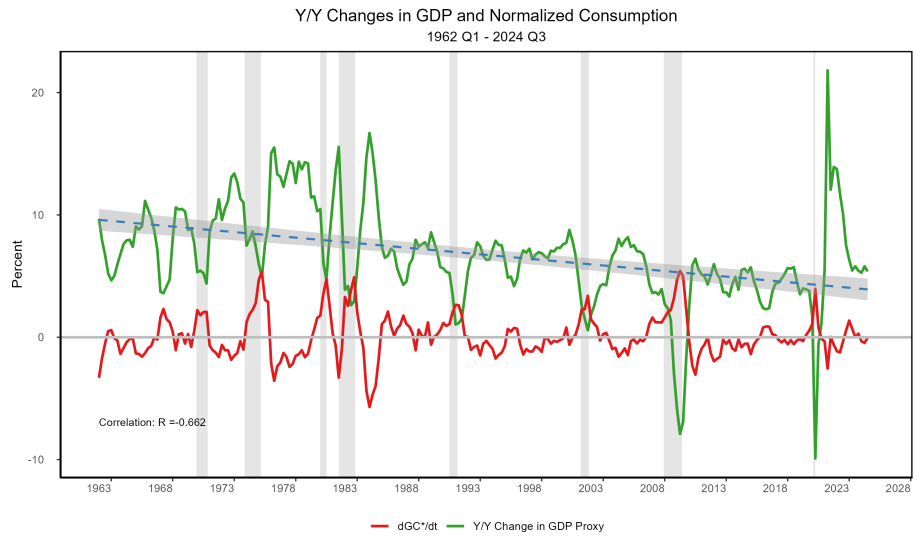

Figure 25: *Y/Y Changes in GDP and Normalized Gross Consumption:* This chart shows that while the peaks of dGC^*/dt correspond temporally with the troughs of the GDP Proxy (dY/dt), they do not reach the CGR unless the troughs reach or fall below the Zero Line.

While Figure 25 reveals discrepancies caused by inflation, Figure 26 (*Y/Y Changes in Real GDP and Normalized Gross Consumption*) demonstrates that these discrepancies virtually disappear when substituting real income derivatives for their nominal counterparts.

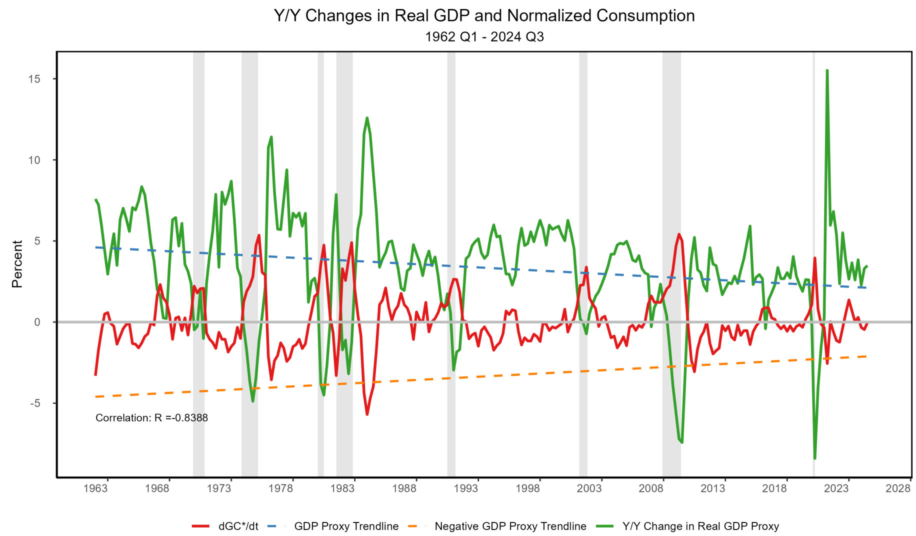

Figure 26: *Y/Y Changes in Real GDP and Normalized Consumption:* This chart shows that the peaks of dGC^*/dt correspond temporally with the troughs of the real GDP Proxy (dY/dt) and that they tend to reach the CGR when the troughs reach or fall below the Zero Line. The chart also shows that dC^*/dt is bounded by the positive and negative *Y/Y Change in GDP Proxy* trend lines, which represent the CGR.

Figure 26 also shows the near perfected boundedness of dGC^*/dt by the positive and negative GDP Proxy rate of change trendlines (dY/dt), which represent the CGR and its opposite, once again confirming conformity with the Augmented NIE Model Business Cycle.

As would be expected from the Zero-Sum Dynamic, Figure 27 (*Real Normalized Investment vs Central Growth Rate*) shows that the peaks of dNI^*/dt align with the CGR as represented by the real GDP Proxy trendline.

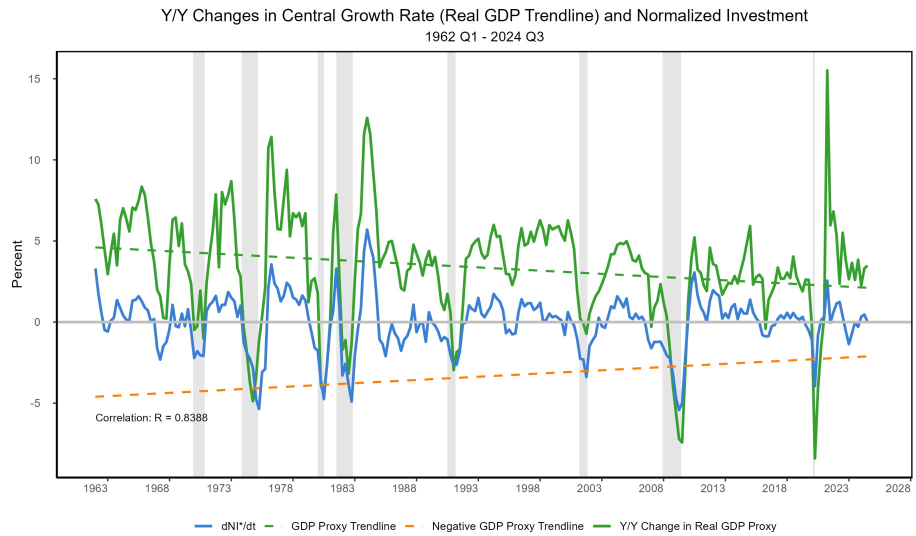

Figure 27: *Real Normalized Investment vs Central Growth Rate:* This chart shows that the peaks of income-normalized Net Private Domestic Investment derivative (dNI^*/dt) approach the real CGR. The chart also shows that dI^*/dt is bounded by the positive and negative Y/Y Change in GDP Proxy trend lines, which represent the CGR.

We cannot overstate the significance of these findings. The alignment of $dGC^{\#}/dt$ with the CGR demonstrates that consumption growth strives toward optimized growth, dynamically balancing income and consumption over time to maintain structural stability within the system. That the CGR also caps the peaks of dC^*/dt and dI^*/dt validates the boundedness of the observed oscillations in the business cycle and confirms that the principle of marginal utility plays a significant role in shaping them. While inflation introduces variability into nominal data, using real data reveals how closely consumption growth tracks theoretical predictions, underscoring its adaptive efficiency in responding to real-world economic conditions. Our results also confirm the effectiveness of the income-normalized model by showing how income-normalized models effectively capture real-world economic dynamics, offering insights into cyclical behavior that may be obscured by inflation or other distortions in nominal data.

The Zero Line as a Benchmark. To indicate the full importance of the Zero Line as a benchmark, we must demonstrate the equivalence $dI^*/dt = dI^{\#}/dt$, which was first mentioned above in Section 7.7.2 (*The Augmented NIE Model Business Cycle*) as part of the exposition on our

Augmented NIE Model Business Cycle. The equivalence demonstrates that income-normalized net investment growth in nominal terms equals income-normalized net investment growth in real terms, which implies that both variables oscillate around the Zero Line.

We derive this relationship as follows:

Step 1: Start with the following known relationships:

From Equation (14), establishing the relationship among the Central Growth Rate, income-normalized income growth, and income-normalized gross consumption growth:

$$CGR = \frac{dC^{\#}}{dt} = \frac{dY}{dt} + \frac{dC^{*}}{dt}$$

From Equation (10), the Zero-Sum Dynamic, ensuring that income-normalized changes in consumption and investment are inversely related:

$$\frac{dC^{*}}{dt} = -\frac{dI^{*}}{dt}$$

From Equation (9), the basic NIE Model derivative equation:

$$\frac{dY}{dt} = \frac{dC^{\#}}{dt} + \frac{dI^{\#}}{dt}$$

Step 2: Rearrange Equation (9) as follows:

$$\frac{dI^{\#}}{dt} = \frac{dY}{dt} - \frac{dC^{\#}}{dt}$$

Step 3: Derive $dI^{\#}/dt$ in terms of dC^{*}/dt with the following steps:

Step 3a: Substitute the CGR Proposition into the equation for $dI^{\#}/dt$:

$$dI^{\#}/dt = dY/dt - CGR$$

Step 3b: Replace the CGR with its definition Using the CGR definition from Equation (14):

$$dI^{\#}/dt = dY/dt - (dY/dt + dC^{*}/dt)$$

Step 3c: Simplify to arrive at the derivation of $dI^{\#}/dt$ in terms of dC^{*}/dt:

$$dI^{\#}/dt = -dC^{*}/dt$$

Step 4: Substitute the Zero-Sum Dynamic from Equation (10) ($dC^{*}/dt = -dI^{*}/dt$):

$$dI^{\#}/dt = dI^{*}/dt \tag{19}$$

Figure 28 (*Comparison of Nominal Y/Y Changes in Income-Adjusted Investment and Normalized Investment*) illustrates the oscillation of income-normalized net investment growth ($dNI^{\#}/dt \approx dNI^{*}/dt$) around the Zero Line across the Full Study Period. The close alignment of the trendlines and the high correlation of the two series supports the validity of Equation (19).

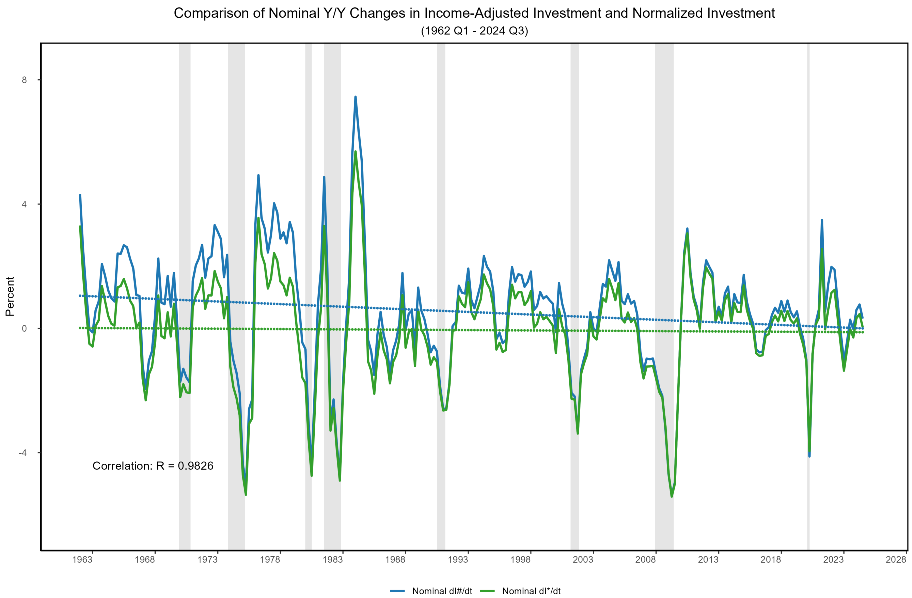

Figure 28: *Comparison of Nominal Y/Y Changes in Income-Adjusted Investment and Normalized Investment:* This chart shows the close alignment of the nominal income-adjusted Net Private Domestic Investment (NDPI) derivative ($NI^{\#}/dt$) and income-normalized NDPI derivative (dNI^{*}/dt) trendlines and the high correlation of the two series.

Figure 29 (*Excess of GDP over CGR vs Y/Y Change in Normalized Net Income*) effectively aligns the CGR (represented by the GDP Proxy trendline) with the Zero Line to show that the oscillations of GDP (dY/dt) and income-normalized net investment ($dNI^{\#}/dt$) are close in alignment and magnitude, supporting the relation of the two variables in our Augmented NIE Model Business Cycle.

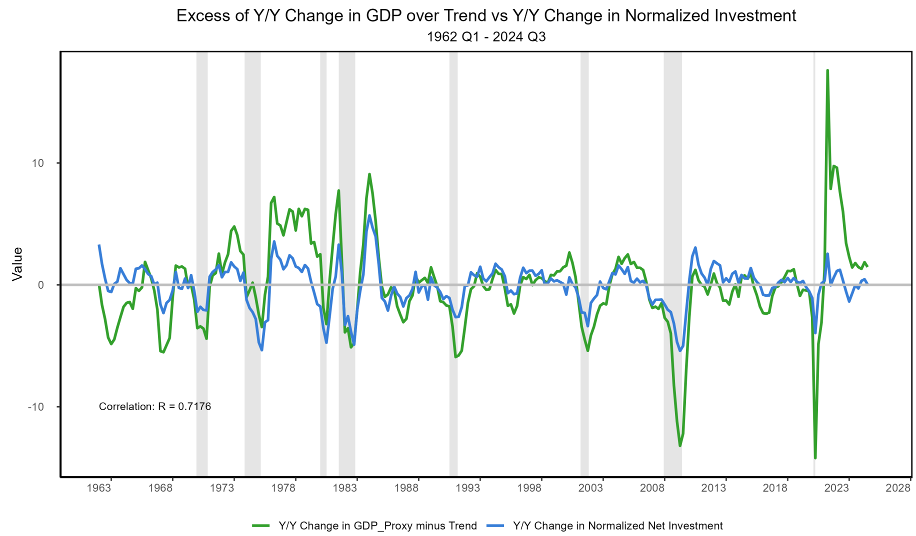

Figure 29: *Excess of Y/Y Change in GDP over Trend vs Y/Y Change in Normalized Investment:* This chart shows the close alignment of the GDP Proxy (dY/dt) and income-normalized Net Private Domestic Investment (dNI^*/dt) derivative trendlines and the high correlation of the two series.

Not only do Figures 22 through 29 confirm the alignment of the Augmented NIE Variables in our Augmented NIE Model Business Cycle, but the combined effect of the structural dynamics shown in the figures and the model is profound. The system achieves the dual efficiency of consumption and investment, to which we will now turn.

8 Dual Pillars of the Endogenous Business Cycle

The dual efficiency of the economy demonstrates its remarkable adaptability. Consumption growth optimally aligns with the Central Growth Rate (CGR), while investment growth dynamically adjusts to realize potential output. These dynamics are encapsulated in the relationships CGR $\approx dC^{\#}/dt$ and $dI^*/dt \approx dI^{\#}/dt$, which constitute the dual pillars of the endogenous business cycle. Together, they reveal how consumption functions as the primary driver of economic well-being, while investment serves as the mechanism that converts potential economic returns into realized growth, guided by the time preference for consumption (R). Consumption optimally tracks income at the CGR, while investment dynamically adjusts to sustain output at the optimized level dictated by the relation between potential return on investment (PRI) and the time preference for consumption (R). The interplay between PRI and R ensures that the system remains structurally efficient and capable of sustaining long-term growth while responding to internal and external pressures.

8.1 Consumption and Investment Efficiency

The Central Growth Rate (CGR) serves as the benchmark for consumption efficiency, which, within the Augmented NIE Model Business Cycle, refers to the optimal allocation of resources to maximize long-term sustainable growth while minimizing deviations from the CGR. The CGR captures the long-term trend rate of economic expansion. It is important to note that while the model assumes a constant CGR, the real-world CGR is a dynamic benchmark that the economy adapts to over time. The relationship $dC^{\#}/dt \approx CGR$ demonstrates that income-adjusted consumption growth closely tracks the CGR, ensuring the optimal alignment of income and consumption growth across business cycles. Even in the presence of external disruptions such as inflation or financial shocks, this dynamic alignment minimizes waste and maximizes the utility of marginal income. Figure 24 (*Comparison of Consumption with the Central Growth Rate*) demonstrates this strong alignment by showing a near-perfect correlation between the income-adjusted gross consumption growth rate ($dC^{\#}/dt$) and the imputed CGR across the Full Study Period. This is further reinforced by Figure 25 (*Y/Y Changes in GDP and Normalized Gross Consumption*), which shows how peaks in the income-normalized consumption growth rate (dC^{*}/dt) consistently coincide with troughs in income growth, indicating the counter-cyclical nature of consumption.

Investment efficiency complements consumption efficiency by dynamically adjusting investment growth to match the economy's potential output. Equation (19) ($\frac{dI^{*}}{dt} \approx \frac{dI^{\#}}{dt}$) demonstrates the alignment between income-normalized and income-adjusted investment growth, both oscillating around the Zero Line. This line acts as a pivotal turning point where the rate of change of investment briefly reaches zero, signaling a transition between phases of investment or consumption. The equivalence of $\frac{dI^{*}}{dt}$ and $\frac{dI^{\#}}{dt}$ is central to the Augmented National Income Equilibrium (NIE) Model Business Cycle, but it does not directly ensure an efficient allocation of resources. Instead, the dynamic interplay between the potential return on investment (PRI) and the time preference for consumption (R) ensures the efficient allocation of income.

As the PRI-R Differential ($PRI - R$) increases, investment accelerates, fostering economic expansion. Conversely, a decrease in the PRI-R Differential leads to a slowdown in investment, prompting a reallocation of resources towards consumption, which is the primary objective of economic activity. The system dynamically reallocates resources to balance the trade-offs between immediate consumption and investment for future consumption and growth, continually optimizing within an ever-changing context. By maintaining structural stability and facilitating persistent oscillations, the CPE demonstrates an ability for self-regulation, ensuring long-term stability and resilience through business cycles.

8.2 The Mediating Role of R

As shown in Section 4.5 (*Time Preference as the Income Allocation Mediator*), R mediates the allocation of income between consumption and investment, dynamically balancing present consumption with potential future returns. The PRI-R Differential adjusts to ensure investment efficiency, while R sets the baseline for current consumption demand, serving as the mediator for income allocation. By managing the relationships among PRI, dY/dt, and $dI^{\#}/dt$, R keeps the system adaptable, supporting long-term growth while responding to cyclical changes. This adjustment enables continuous market clearing, optimizing both immediate consumption and future investment opportunities to sustain structural efficiency across business cycles.

8.3 Implications for the Endogenous Business Cycle

The dual pillars of $dC^{\#}/dt \approx CGR$ and $dI^{*}/dt \approx dI^{\#}/dt$, when combined with the mediating role of R, provide a structured framework for understanding the endogenous business cycle, in which economic fluctuations arise naturally from the systematic interaction of the Augmented NIE Augmented NIE Model Variables rather than from exogenous shocks. The Augmented NIE Model Business Cycle framework not only offers a foundation for further study but can also inform policy by revealing how fiscal and monetary interventions interact with the business cycle's structural dynamics. By aligning policy with these underlying forces, economic resilience can be enhanced while mitigating the risks of disruptive intervention.

9 Summary and Broader Implications

9.1 Recap of Core Findings

The empirical analysis in this study strongly supports the Augmented NIE Model Business Cycle as a robust framework for understanding economic oscillations. By defining the GDP Proxy, which is constructed from consumption ($GPCE$) and investment ($NDPI$) proxies, as a reliable metric aligned with National Income Accounting, we laid the groundwork for confirming the model's core tenets: that the goal of economic activity is consumption and that sustained growth through business cycles depends on consumption and investment expanding in proportion to income.

We confirmed several fundamental relationships among the Augmented NIE Augmented NIE Model Variables.

First, the Zero-Sum Dynamic, expressed as

$$dC^*/dt = -dI^*/dt,$$

demonstrates that income-normalized consumption and investment growth are cyclically opposed, driving the oscillatory nature of the business cycle, and this relationship is the driving force behind the oscillatory nature of the business cycle..

Second, the relationship

$$dC^{\#}/dt \approx CGR$$

establishes that income-adjusted gross consumption growth aligns closely with the CGR, which serves as both a benchmark for optimal consumption and a cap on consumption and investment growth, while also dynamically adjusting to track real-world fluctuations in the CGR.

Third, the equivalence

$$dI^*/dt \approx dI^{\#}/dt$$

confirms that income-normalized and income-adjusted investment growth oscillate around the Zero

Line in response to the changing PRI-R Differential, which is the mechanism that reallocates income between consumption and investment.

Fourth, consumption growth dC^*/dt explains recognized income-normalized interest rate proxies, reinforcing its role as a determinant of the pure interest rate R.

Finally, the Consumption-Normalized Identity of Income and Investment, given by

$$\frac{d(Y/C)}{dt} = \frac{d(I/C)}{dt},$$

empirically confirms that changes in income relative to consumption necessarily translate into equivalent changes in investment relative to consumption, reinforcing R as the dynamic regulator of income allocation, and emphasizing that these fluctuations are a part of the endogenous business cycle and not a market failure.

The results of the study demonstrate that stable disequilibrium, not external shocks, is the fundamental driver of business cycles. The cyclical fluctuations of the economy arise from the internal dynamics of income (dY/dt), consumption ($dC^{\#}/dt$), and investment ($dI^{\#}/dt$) as they continuously adjust, with the pure interest rate (R) mediating these changes. This continuous adjustment process is how the economy strives to maximize the potential return on investment (PRI) within the constraints of time preference.

In summary, the business cycle emerges from the economy's endogenous efforts to balance consumption and investment, and the pure interest rate, R, mediates this process. These cycles are not a failure, but a natural part of the market's operation and are driven by the internal dynamics of the market rather than external shocks.

9.2 Conclusion

The Augmented National Income Equilibrium Model Business Cycle offers a novel perspective on economic fluctuations, differing significantly from traditional business cycle theories. Traditional theories often assume that economies tend towards a stable equilibrium and that cycles are disruptions caused by external factors such as monetary shocks or technological changes. In contrast, the

Augmented NIE Model Business Cycle proposes that business cycles are endogenous and essential for maximizing economic efficiency, arising from the dynamic interplay of income, consumption, and investment, mediated by the pure interest rate (R) and driven by the potential return on investment (PRI). Here are the key ways in which the Augmented NIE Model Business Cycle is distinguished from traditional theories:

First, traditional theories often view cycles as deviations from a stable equilibrium, while the Augmented NIE Model Business Cycle posits that economies operate in a continuous state of disequilibrium. This disequilibrium arises from the ongoing allocation of income between consumption and investment.

Second, traditional theories frequently cite external factors like monetary policy, technological shifts, or changes in aggregate demand as the primary drivers of business cycles. The Augmented NIE Model Business Cycle, however, identifies the constant tension between investment and consumption, mediated by the pure interest rate (R), as the core driver.

Third, traditional theories often consider interest rates a tool to achieve a steady-state economy, focusing on maintaining a "natural rate". In contrast, the Augmented NIE Model Business Cycle sees the pure interest rate (R) as an endogenous outcome of business cycle dynamics, aligning with the Central Growth Rate (CGR) to regulate resource allocation over time.

Fourth, traditional models often view cycles as anomalies that need correction, while the Augmented NIE Model Business Cycle views them as an inherent and efficient part of economic dynamics. The model demonstrates that these cycles occur even without external shocks, highlighting their self-regulating nature.

Finally, traditional theories often suggest that government intervention can dampen or accelerate business cycles, while the Augmented NIE Model Business Cycle implies that such interventions may disrupt the economy's self-regulating efficiency.

Key relationships within the Augmented NIE Model Business Cycle that contrast with traditional theories include: (i) the Zero-Sum Dynamic ($dC^*/dt = -dI^*/dt$), showing that income-normalized consumption and investment growth are cyclically opposed; (ii) the alignment of income-adjusted gross consumption growth with the Central Growth Rate (CGR) ($dC^{\#}/dt \approx CGR$) which acts as a benchmark for optimal consumption; and (iii) the fact that the pure interest rate (R) is an endogenous factor that mediates the allocation of income between consumption and investment. Moreover, the Consumption-Normalized Identity of Income and Investment ($d(Y/C)/dt = d(I/C)/dt$) confirms that changes in income relative to consumption translate into equivalent changes in investment relative to consumption, reinforcing R's role as a dynamic regulator. The equivalence $dI^*/dt = dI^\#/dt$ also confirms that income-normalized and income-adjusted investment growth oscillate around the Zero Line, demonstrating the dynamic allocation of resources between investment and consumption.

The Augmented NIE Model Business Cycle illustrates that business cycles are not deviations from equilibrium but are part of a self-regulating process inherent in market economies. The cycles are the result of the economy's internal efforts to balance consumption and investment, mediated by the pure interest rate, R. The model challenges the traditional focus on external shocks as the primary drivers of economic fluctuations, instead emphasizing the internal dynamics of income, consumption, and investment. The Augmented NIE Model Business Cycle framework can provide a basis for more informed economic policy that respects the market's self-regulating mechanisms and the limitations of intervention.

10 Policy Implications: A Refined Perspective Centered on Cyclical Stability

The validation of our theoretical framework reinforces two core economic principles: that the primary aim of economic activity is consumption and that sustained long-term growth necessitates that consumption and investment align with income growth. Our analysis discloses that the income-normalized cyclical behavior of the economy demonstrates a self-sustaining and inherently stable pattern that tends toward complete realization of economic potential (PRI) in output growth (MY). Empirical findings, such as the high pseudo-R^2 values and consistent correlations among derivatives, underscore that business cycles are both naturally occurring and integral to economic dynamics, compelling the conclusion that fiscal and monetary policies should prioritize restraint, recognizing that excessive intervention may disrupt this intrinsic stability. It is also important to acknowledge that central bank policy does not operate in a vacuum and must take into account fiscal policy; specifically, in an environment of profligate spending, the central bankers' task becomes more complex, given the potential for that spending to disrupt or obscure the naturally occurring dynamics of our model. Here are some recommendations.

Embracing Natural Economic Adjustments: Governments should carefully evaluate possible interventions in light of economic cycles' demonstrated bound yet non-decaying nature, as revealed by our model and the cyclical forces that bind it. The empirical data suggests that attempts to suppress these natural oscillations disrupt the inherent efficiency of the cyclical process, potentially leading to unintended economic consequences. Economic policy should prioritize fostering an environment that allows natural adjustments, recognizing that cyclical fluctuations are a fundamental and beneficial part of economic health, which tends toward maximization of economic potential, as demonstrated by our analysis. Rather than attempting to suppress these cycles, policymakers should respect the intrinsic balancing mechanisms of the market, which promote sustainable growth without excessive external intervention, and focus on mitigating the excesses that cause severe disruption and not on preventing the cycles from existing.

Caution Against Disruptive Interventions: Intervening to preempt natural economic adjustments can add inefficiencies to the underlying economic structures and the realization of the economy's potential and may exacerbate rather than mitigate instability. Policy measures to avoid the natural consequences of cyclical activity often involve artificially induced wealth redistributions. While such redistributive actions may be necessary in cases of significant economic distress, they should be cautiously approached and justified within the broader political context. When deemed desirable, these redistributions should occur transparently after ratification through established democratic processes to ensure public support and legitimacy, with direct reference to our understanding of diminishing marginal utility and to the understanding that all redistribution policies have a cost to the overall economy.

Principles for Policy Development

Policymakers should prioritize stability and market function by allowing the economy's self-correcting mechanisms to operate. This requires maintaining prudent monetary policy, safeguarding property rights, supporting competition, and ensuring a stable regulatory framework, as defined by the components of our model. It is important to recognize that, within our framework, the CGR defines the sustainable long-term growth rate of the economy. Consumption and investment must adhere to its principles to prevent disequilibrium.

In formulating economic policies, it is essential to distinguish between short-term cyclical variations and long-term secular shifts driven by technological or demographic changes. Addressing secular shifts through structural policies is often more effective than cyclical interventions that attempt to counteract natural economic adjustments. Moreover, policymakers should not assume that any given intervention will necessarily reduce deviations from the CGR.

If intervention is warranted, it should be clearly defined, temporary, and guided by transparent objectives to avoid undermining market expectations and introducing long-term distortions. Effective communication of an intervention's intended benefits and limitations is crucial to maintaining public trust and market stability.

Finally, fixed policy benchmarks should be reevaluated. Our analysis challenges the notion of a static natural rate of interest (*r-star*) as a reliable balancing point for economic growth and inflation. The cyclical oscillations of consumption and investment necessitate continuous adjustment rather than adherence to a fixed benchmark. Instead of imposing a rigid *r-star*, policymakers should adopt a more adaptive framework that accounts for evolving economic conditions and does not arbitrarily constrain R. Our findings suggest that a dynamic, market-driven "natural rate" of interest—best represented as $dC^*/dt + dY/dt$, closely aligned with the Central Growth Rate—emerges naturally within an unconstrained system. The central bank's role, therefore, should be to facilitate conditions that allow the market to self-discover this rate rather than attempting to dictate it through intervention.

Conclusion Policymakers should ground their strategies in an understanding that the economy's income-normalized cyclical behavior reflects a natural and resilient stability. Maintaining resilience requires policies that respect these natural cyclical dynamics. Recognizing the consumption-investment balance as central to long-term stability allows for a policy framework that respects the self-regulating tendencies of the market while acknowledging the need for targeted responses during significant disruptions.

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APPENDIX A: NORMALIZATION AND UNNORMALIZATION METHODOLOGY

A. Introduction to Normalization: The normalization process in our model serves two purposes: (1) to remove the effects of income levels from our variables, allowing us to observe the pure oscillation between consumption and investment around income, and (2) to express these variables as proportions of income rather than absolute values, showing us how income is being allocated between consumption and investment at any point. Figure 20 (*Normalized Augmented NIE Model Business Cycle*) is an example of the power of the normalization process. It shows how the income-normalized variables dGC^*/dt and dNI^*/dt oscillate inversely around the Zero Line, while PRI^* remains aligned with the Zero Line indicating efficient market conditions in which the potential return on investment is (almost) totally realized in output within the constraint of time preference.

B. Basic Normalization Definitions: For any variable X in our model, we define five related quantities.

Step 1: Define the raw change in X. The difference between two measured values of X is given by:

$$\Delta X = X_{T1} - X_{T0},$$

where $T0$ is the first measuring time and $T1$ is the second measuring time.

Step 2: Define the rate of change of X. Expressing the rate of change as a percentage of its initial level:

$$\frac{dX}{dt} = 100 \cdot \frac{\Delta X}{X_{T0}}.$$

Step 3: Define the income-share adjusted change in X. Relative to income Y, the adjustment is given by:

$$\frac{dX^{\#}}{dt} = \frac{dX}{dt} \cdot \frac{X}{Y}.$$

Step 4: Define the rate of change of the share of X in income. The normalized form of X adjusted for its current share is:

$$\frac{dX^*}{dt} = \frac{d(X/Y)}{dt} \cdot \frac{X}{Y}.$$

Rewriting, this simplifies to:

$$\frac{dX^*}{dt} = \frac{X}{Y^2} \cdot \frac{dX}{dt} - \frac{X^2}{Y^3} \cdot \frac{dY}{dt}.$$

C. The Unnormalization Process: To convert from income-normalized variables $\frac{dX^*}{dt}$ and $\frac{dX^\#}{dt}$ back to their unnormalized forms (X):

Step 1: Restate $\frac{dX^*}{dt}$ *in terms of* $\frac{dX}{dt}$.

$$\frac{dX}{dt} = \frac{\frac{dX^*}{dt}}{\frac{X}{Y}} + X \cdot \frac{dY}{dt}.$$

Step 2: Restate $\frac{dX^\#}{dt}$ *in terms of* $\frac{dX}{dt}$.

$$\frac{dX}{dt} = \frac{dX^\#}{dt} \cdot \frac{Y_{T1}}{X_{T1}}.$$

Step 3: Recover X *from* $\frac{dX^\#}{dt}$.

$$X_{T1} = X_{T0} + \sum_{t=T0}^{T1} \left(\frac{dX^\#}{dt} \cdot \frac{Y_t}{X_t} \cdot \Delta t \right),$$

where X_{T0} is the initial value of X at time T_0, and the sum accumulates the changes in X over discrete time periods Δt.

D. Special Case of PRI^*: PRI differs fundamentally from consumption and investment in our normalization process. While C and I are components of Y that must sum to Y, PRI represents a rate of return that exists independently of Y.

Step 1: Normalize PRI relative to the economy's growth rate.

$$PRI^* = PRI - \frac{dY}{dt}.$$

This transformation adjusts the potential return on investment (PRI) to reflect its value relative to the overall growth rate of the economy.

Step 2: Interpret the equilibrium condition of PRI^*. When $PRI^* = 0$, it follows that:

$$PRI = \frac{dY}{dt}.$$

This equality implies that the potential return on investment is being fully realized as income growth, a condition expected in an efficient market where resources are optimally allocated.

Step 3: Empirical validation in Figure 20. The alignment of the green line (PRI^*) with the Zero Line throughout the business cycle suggests that market efficiency ($PRI = dY/dt$) is maintained, even as the economy oscillates between consumption-led and investment-led growth. Moreover, the cyclical movement of dGC^*/dt and dNI^*/dt around zero supports this dynamic equilibrium.

APPENDIX B: DERIVATION OF THE RATE OF CHANGE OF INCOME-NORMALIZED CONSUMPTION

This appendix provides a step-by-step derivation of the equation for the rate of change of income-normalized consumption (dC^*/dt), as used in Section 7.7:

$$dC^*/dt = \frac{dY}{dt} \cdot \frac{(1 - C^*)}{Y}.$$

This derivation is based on the foundational equations of the Augmented NIE Model and the assumption that all additional income is allocated to consumption.

Step 1: Define income-normalized consumption.

$$C^* = \frac{C}{Y}.$$

Step 2: Derive the general rate of change formula.

Step 2a: Use the generic equation for the rate of change of any variable.

$$\frac{dX^*}{dt} = \frac{1}{Y}\left(\frac{dX}{dt} - \frac{X}{Y} \cdot \frac{dY}{dt}\right).$$

Step 2b: Apply this formula to determine the rate of change of income-normalized consumption (dC^*/dt).

$$\frac{dC^*}{dt} = \left(\frac{dC}{dt}/Y\right) - \left(\frac{C}{Y^2} \cdot \frac{dY}{dt}\right).$$

Step 3: Assume that all increases in income (ΔY) go to increases in consumption (ΔC), so that the rate of change of consumption is equal to the rate of change of income ($dC/dt = dY/dt$).

Step 4: Substitute dY/dt for dC/dt into Step 2b.

$$\frac{dC^*}{dt} = \left(\frac{dY}{dt}/Y\right) - \left(\frac{C}{Y^2} \cdot \frac{dY}{dt}\right).$$

Step 5: Factor out dY/dt.

$$\frac{dC^*}{dt} = \frac{dY}{dt}\left(\frac{1}{Y} - \frac{C}{Y^2}\right).$$

Step 6: Combine the two terms.

$$\frac{dC^*}{dt} = \frac{dY}{dt} \cdot \frac{Y - C}{Y^2}.$$

Step 7: Replace $(Y - C)$ with I.

$$\frac{dC^*}{dt} = \frac{dY}{dt} \cdot \frac{I}{Y^2}.$$

Step 8: Express income-normalized investment (I^*) as I/Y, so that $I = I^* \cdot Y$. Substituting into Step 7:

$$\frac{dC^*}{dt} = \frac{dY}{dt} \cdot \frac{I^*Y}{Y^2}.$$

Step 9: Simplify the expression.

$$\frac{dC^*}{dt} = \frac{dY}{dt} \cdot \frac{I^*}{Y}.$$

Step 10: Express Equation 9 in terms of C^*.

$$\frac{dC^*}{dt} = \frac{dY}{dt} \cdot \frac{(1 - C^*)}{Y}.$$

Summary of the Steps. This derivation demonstrates how, starting from the basic definition of consumption share, we arrive at the final equation. It emphasizes that this equation is a direct consequence of our definitions and assumptions, and that it does not exist in isolation from the broader framework of our model.

APPENDIX C: SUMMARY OF Augmented NIE Model EQUATIONS

1. $Y = C + I$

   Representation: Income (Y) comprises Consumption (C) and Investment (I).

   Meaning: Defines total income in terms of its basic components.

2. $\text{PRI} = R + \frac{dI^{\#}}{dt}$

   Representation: The potential return on investment (PRI) equals the real interest rate (R) plus the income-share adjusted change in investment.

   Meaning: Establishes the relationship between the potential return on investment, the real interest rate, and the change in investment.

3. $\Delta Y = \Delta C + \Delta I$

   Representation: The change in income (ΔY) is the sum of the changes in consumption (ΔC) and investment (ΔI).

   Meaning: Captures income changes directly tied to consumption and investment shifts.

4. $\frac{dY}{dt} = 100 \cdot \frac{\Delta Y}{Y_{T0}}$

   Representation: The growth rate of income equals 100 times the change in income divided by the initial value of income.

   Meaning: Formalizes the calculation of the income growth rate as a percentage change.

5. $\frac{dC}{dt} = 100 \cdot \frac{\Delta C}{C_{T0}}$

   Representation: The growth rate of consumption equals 100 times the change in consumption divided by the initial value of consumption.

   Meaning: Formalizes the calculation of the consumption growth rate as a percentage change.

6. $\frac{dI}{dt} = 100 \cdot \frac{\Delta I}{I_{T0}}$

   Representation: The growth rate of investment equals 100 times the change in investment divided by the initial value of investment.

   Meaning: Formalizes the calculation of the investment growth rate as a percentage change.

7. $\frac{dY}{dt} = \frac{dC}{dt} + \frac{dI}{dt}$

Representation: The growth rate of income is the sum of the growth rates of consumption and investment.

Meaning: Illustrates how income growth is driven by changes in consumption and investment.

8. $\frac{dY/dt}{Y} = \left(\frac{dC/dt}{C} \cdot \frac{C}{Y}\right) + \left(\frac{dI/dt}{I} \cdot \frac{I}{Y}\right)$

Representation: The growth rate of income is expressed as a weighted average of the growth rates of income-share adjusted consumption and investment.

Meaning: Shows how income growth is driven by the relative shares and growth rates of income-share adjusted consumption and investment.

9. $\frac{dY}{dt} = \frac{dC^{\#}}{dt} + \frac{dI^{\#}}{dt}$

Representation: The growth rate of income expressed through income-share adjusted consumption and investment rates.

Meaning: Offers an income-relative perspective on growth dynamics.

10. $\frac{dC^*}{dt} = -\frac{dI^*}{dt}$

Representation: Zero-Sum Dynamic, where changes in income-normalized consumption and investment shares offset each other.

Meaning: Reflects the inverse relationship between income-normalized consumption and investment shares.

11. $R = \mathrm{PRI} - \frac{dI^{\#}}{dt}$

Representation: Pure interest rate (R) equals the potential return on investment (PRI) minus the rate of change of income-share adjusted investment.

Meaning: Shows how the pure interest rate determines the relationship between potential return on investment and the rate of income-share adjusted investment.

12. $R^* = \mathrm{PRI}^* - \frac{dI^*}{dt}$

Representation: The income-normalized interest rate (R^*) equals the income-normalized potential return on investment (PRI^*) minus the income-normalized change in investment.

Meaning: Shows how the income-normalized pure interest rate determines the relationship between income-normalized potential return on investment and the rate of income-normalized investment.

13. $\text{PRI} - R = \frac{dY}{dt} - \frac{dC^{\#}}{dt}$

Representation: The PRI-R Differential defines the relationship between income growth and income-share adjusted consumption growth.

Meaning: Links growth dynamics with investment opportunities.

14. $\text{CGR} = \frac{dY}{dt} + \frac{dC^*}{dt} = \frac{dC^{\#}}{dt}$

Representation: The Central Growth Rate is the sum of income growth and income-normalized consumption growth, and is also equal to income-adjusted consumption growth.

Meaning: Defines a central benchmark for balanced economic growth and signifies that income-adjusted consumption growth tracks the Central Growth Rate.

15. $R = \text{CGR}$

Representation: The real interest rate equals the central growth rate.

Meaning: Establishes the equivalence between the real interest rate and the central growth rate.

16. $\frac{d(Y/C)}{dt} = \frac{d(I/C)}{dt}$

Representation: The growth rate of consumption-normalized income equals the growth rate of consumption-normalized investment.

Meaning: Shows that consumption is the mediator between income and investment.

17. $PRI - R = 100 \cdot \frac{d(I/C)}{dt}$

Representation: The PRI-R Differential is equal to $100 \cdot \frac{d(I/C)}{dt}$.

Meaning: Expresses the equal correspondence of the PRI-R Differential to the percent rate of change of the investment-normalized rate of consumption.

18. $\text{CGR} = \frac{dY}{dt} + \frac{dC^*}{dt} = \frac{dC^{\#}}{dt}$

Representation: The Central Growth Rate is the sum of income growth and income-normalized consumption growth, and is also equal to income-adjusted consumption growth.

Meaning: Defines a central benchmark for balanced economic growth and signifies that income-adjusted consumption growth tracks the Central Growth Rate.

19. $\dfrac{dI^{\#}}{dt} = \dfrac{dI^{*}}{dt}$

Representation: The income-share adjusted change in investment equals the income-normalized change in investment.

Meaning: Establishes the equivalence between the normalized and unnormalized investment growth rates.

